

DIRECT VARIATION

Recall that direct variation is a linear function of the form $y = kx$, where k is the nonzero constant of variation.

For each function, determine whether y varies directly with x . If so, find the constant of variation and write the equation.

$k = \frac{y}{x}$

1.

x	1	2	3
y	1	4	9

No

$k=1 \quad k=2 \quad k=3$

2.

x	-1	1	3
y	-3	3	9

$k=3$

Yes

In each exercise, y varies directly with x . Find the missing value.

3. If $y=3$ when $x=2$, find x when $y=5$.

$y=kx$
 $3=k(2)$

① $k = \frac{3}{2}$

② $y = \frac{3}{2}x$

③ $5 = \frac{3}{2}x \cdot \frac{2}{3}$
 $x = \frac{10}{3}$

4. If $y=-4$ when $x=\frac{1}{2}$, find y when $x=\frac{2}{3}$.

$y=kx$

① $k = -8$

② $y = -8x$

③ $y = -8(\frac{2}{3})$
 $y = -\frac{16}{3}$

$2 \cdot -4 = k(\frac{1}{2}) \cdot 2$

5. If $y=-14$ when $x=-7$, find x when $y=22$.

$\frac{-14}{-7} = \frac{k(-7)}{-7}$
 $k=2$

$y=2x$
 $\frac{22}{2} = \frac{2x}{2}$
 $x=11$

INVERSE VARIATION

A function of the form $y = \frac{k}{x}$ or $xy = k$, where $k \neq 0$.

Suppose that x and y vary inversely. Write a function that models each inverse variation.

6. $(3, -5)$

$y = \frac{k}{x}$

① $k = -15$

$3 \cdot -5 = \frac{k}{3} \cdot 3$

② $y = \frac{-15}{x}$

7. $(0.3, 1.4)$

$y = \frac{k}{x}$

① $k = .42$

$0.3 \cdot 1.4 = \frac{k}{0.3} \cdot 0.3$

② $y = \frac{.42}{x}$

8. $(7, 4)$

$y = \frac{28}{x}$

Explain why the table below shows an inverse variation.

9.

$xy = k$

x	0.2	0.6	1.2
y	12	4	2

$k = 2.4$

$12 = \frac{k}{0.2}$

COMBINED VARIATION

- Combines direct and inverse variations in more complex relationships

Combined Variation	Equations Form
y varies <u>directly</u> with the square of x	$y = K \cdot x^2$
y varies <u>inversely</u> with the cube of x	$y = \frac{K}{x^3}$
z varies jointly with x and y ↳ like direct	$Z = K \cdot x \cdot y$
z varies jointly with x and y and inversely with w	$Z = \frac{K \cdot x \cdot y}{w}$
z varies directly with x and inversely with the product of w and y	$Z = \frac{K \cdot x}{w \cdot y}$

Write the function that models each relationship. Find z when $x = 4$ and $y = 9$.

10. z varies directly with x and inversely with y . When $x = 6$ and $y = 2$, $z = 15$.

$$\textcircled{1} z = \frac{Kx}{y} \quad 15 = \frac{3K}{2} \quad \textcircled{2} z = \frac{5x}{y}$$

$$15 = \frac{K(6)}{2} \quad K = 5 \quad \textcircled{3} z = \frac{5(4)}{9} = \frac{20}{9}$$

11. z varies jointly with x and y . When $x = 2$ and $y = 3$, $z = 60$.

$$z = 360$$

12. z varies directly with the square of x and inversely with y . When $x = 2$ and $y = 4$, $z = 3$.

$$z = \frac{16}{3}$$

13. z varies inversely with the product of x and y . When $x = 2$ and $y = 4$, $z = 0.5$.

$$z = \frac{K}{x \cdot y} \quad z = \frac{4}{(2)(4)}$$

$$K = 4 \quad z = \frac{1}{9}$$