

**DIRECT VARIATION**

Recall that direct variation is a linear function of the form  $y = kx$  where  $k$  is the nonzero constant of variation.

For each function, determine whether  $y$  varies directly with  $x$ . If so, find the constant of variation and write the equation.

1. 

x	1	2	3
y	1	4	9

 No

$k = \frac{y}{x}$

2. 

x	-1	1	3
y	-3	3	9

 Yes  $y = 3x$

3. 

x	-2	2	5
y	-1	1	2.5

 Yes  $y = \frac{1}{2}x$

In each exercise,  $y$  varies directly with  $x$ . Find the missing value.

4. If  $y = 3$  when  $x = 2$ , find  $x$  when  $y = 5$ .  $3 = k(2)$   $k = \frac{3}{2}$   $5 = \frac{3}{2}(x)$   $x = \frac{10}{3}$

5. If  $y = -4$  when  $x = \frac{1}{2}$ , find  $y$  when  $x = \frac{2}{3}$ .  $-4 = k(\frac{1}{2})$   $k = -8$   $y = -8(\frac{2}{3})$   $y = -\frac{16}{3}$

6. If  $y = -14$  when  $x = -7$ , find  $x$  when  $y = 22$ .  $-14 = k(-7)$   $k = 2$   $22 = 2(x)$   $x = 11$

**INVERSE VARIATION**

A function of the form  $y = \frac{k}{x}$  or  $xy = k$ , where  $k \neq 0$ .

Suppose that  $x$  and  $y$  vary inversely. Write a function that models each inverse variation.

7.  $(3, -5)$       8.  $(0.3, 1.4)$       9.  $(7, 4)$

$-5 = \frac{k}{3}$   $k = -15$   $y = -\frac{15}{x}$

Is the relationship between the variables in each table a direct variation, an inverse variation, or neither? Write functions to model the direct and inverse variations.

10. 

x	0.5	2	6
y	1.5	6	18

 Direct  $y = 3x$

11. 

x	0.2	0.6	1.2
y	12	4	2

 Inverse  $y = \frac{2.4}{x}$

12. 

x	1	2	3
y	2	1	0.5

 Neither

8)  $1.4 = \frac{k}{0.3}$   $k = .42$   $y = \frac{.42}{x}$

9)  $4 = \frac{k}{7}$   $k = 28$   $y = \frac{28}{x}$

**COMBINED VARIATION**

Combines direct and inverse variations in more complicated relationships

Examples of Combined Variations

Combined Variation	Equations Form
$y$ varies directly with the square of $x$	$y = kx^2$
$y$ varies inversely with the cube of $x$	$y = \frac{k}{x^3}$
$z$ varies jointly with $x$ and $y$	$z = kxy$
$z$ varies jointly with $x$ and $y$ and inversely with $w$	$z = \frac{kxy}{w}$
$z$ varies directly with $x$ and inversely with the product of $w$ and $y$	$z = \frac{kx}{wy}$

Write the function that models each relationship. Find  $z$  when  $x = 4$  and  $y = 9$ .

13.  $z$  varies directly with  $x$  and inversely with  $y$ . When  $x = 6$  and  $y = 2$ ,  $z = 15$ .

$z = \frac{kx}{y}$   $15 = \frac{k(6)}{2}$   $k = 5$   $z = \frac{5x}{y}$

14.  $z$  varies jointly with  $x$  and  $y$ . When  $x = 2$  and  $y = 3$ ,  $z = 60$ .

$z = kxy$   $k = 10$   $z = 10(4)(9) = \frac{20}{9}$

15.  $z$  varies directly with the square of  $x$  and inversely with  $y$ . When  $x = 2$  and  $y = 4$ ,  $z = 3$ .

$z = \frac{kx^2}{y}$   $3 = \frac{k(2)^2}{4}$   $k = 3$   $z = \frac{3x^2}{y}$

15.  $z$  varies inversely with the product of  $x$  and  $y$ . When  $x = 2$  and  $y = 4$ ,  $z = 0.5$ .

$z = \frac{k}{xy}$   $z = \frac{4}{xy}$   $z = \frac{3(4)^2}{9}$   
 $0.5 = \frac{k}{2(4)}$   $z = \frac{4}{4(9)}$   $z = \frac{48}{9}$   
 $k = 4$   $z = \frac{4}{36} = \frac{1}{9}$

Find the Missing Variable:

- 1) y varies directly with x. If y = -4 when x = 2, find y when x = -6.  $y = 12$
- 2) y varies inversely with x. If y = 40 when x = 16, find x when y = -5.  $y = -128$
- 3) y varies inversely with x. If y = 7 when x = -4, find y when x = 5.  $y = 7$
- 4) y varies directly with x. If y = 15 when x = -18, find y when x = 1.6.  $y = -4/3$
- 5) y varies directly with x. If y = 75 when x = 25, find x when y = 25.  $x = 25/3$

Classify the following as: a) Direct b) Inverse c) Neither

- 6)  $m = -5p$  **a**
- 7)  $r = \frac{9}{t}$  **b**
- 8)  $d = 4t$  **a**
- 9)  $c = \frac{e}{-4}$  **a**
- 10)  $n = \frac{1}{2}f$  **a**
- 11)  $z = \frac{-2}{t}$  **b**
- 12)  $c = 3v$  **a**
- 13)  $u = \frac{t}{18}$  **a**

What is the constant of variation for the following?

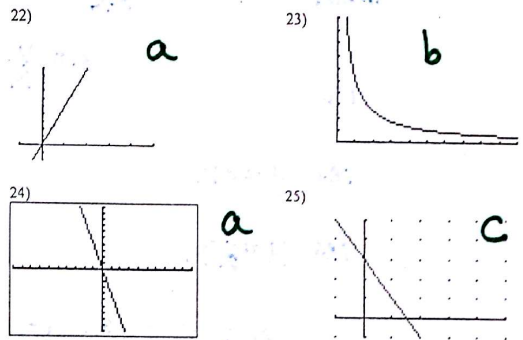
- 14)  $d = 4t$   **$k = 4$**
- 15)  $z = \frac{-2}{t}$   **$k = -2$**
- 16)  $n = \frac{1}{2}f$   **$k = \frac{1}{2}$**
- 17)  $r = \frac{9}{t}$   **$k = 9$**

Answer the following questions.

- 18) If x and y vary directly, as x decreases, what happens to the value of y? **dec**
- 19) If x and y vary inversely, as y increases, what happens to the value of x? **dec**
- 20) If x and y vary directly, as y increases, what happens to the value of x? **inc**
- 21) If x and y vary inversely, as x decreases, what happens to the value of y? **inc**

$y = kx$        $y = \frac{k}{x}$

Classify the following graphs as a) Direct b) Inverse c) Neither



Answer the following questions:

- 26) The electric current I, in amperes, in a circuit varies directly as the voltage V. When 12 volts are applied, the current is 4 amperes. What is the current when 18 volts are applied? **6 amperes**
- 27) The volume V of gas varies inversely to the pressure P. The volume of a gas is 200 cm<sup>3</sup> under pressure of 32 kg/cm<sup>2</sup>. What will be its volume under pressure of 40 kg/cm<sup>2</sup>? **160 cm<sup>3</sup>**
- 28) The number of kilograms of water in a person's body varies directly as the person's mass. A person with a mass of 90 kg contains 60 kg of water. How many kilograms of water are in a person with a mass of 50 kg? **33 1/3 kg**
- 29) On a map, distance in km and distance in cm varies directly, and 25 km are represented by 2cm. If two cities are 7cm apart on the map, what is the actual distance between them? **87.5 km**
- 30) The time it takes to fly from Los Angeles to New York varies inversely as the speed of the plane. If the trip takes 6 hours at 900 km/h, how long would it take at 800 km/h? **6.75 hrs**