

Unit 5 Day 1 AFM Notes: Trigonometric Ratios

Key

Trig Ratios

Sine: $\sin A = \frac{\text{opp}}{\text{hyp}}$

Cosine: $\cos A = \frac{\text{adj}}{\text{hyp}}$

Tangent: $\tan A = \frac{\text{opp}}{\text{adj}}$

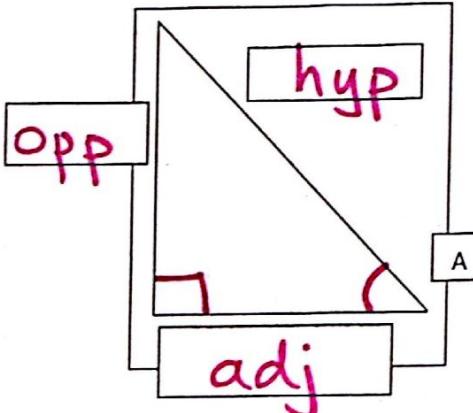
Reciprocal

Inverse Trig Ratios

Cosecant: $\csc A = \frac{1}{\sin A} = \frac{\text{hyp}}{\text{opp}}$

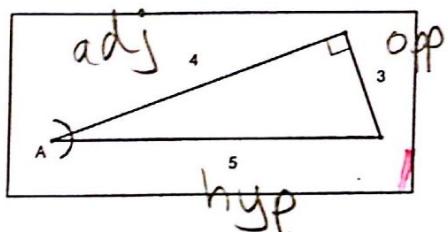
Secant: $\sec A = \frac{1}{\cos A} = \frac{\text{hyp}}{\text{adj}}$

Cotangent: $\cot A = \frac{1}{\tan A} = \frac{\text{adj}}{\text{opp}}$



EX.

1. Find all 6 trigonometric ratios of the following:



$$\sin A = \frac{3}{5}$$

$$\csc A = \frac{5}{3}$$

$$8\sqrt{3}$$

$$\cos A = \frac{4}{5}$$

$$\sec A = \frac{5}{4}$$

$$10\sqrt{4}$$

$$\tan A = \frac{3}{4}$$

$$\cot A = \frac{4}{3}$$

$$2\sqrt{52}$$

$$2\sqrt{26}$$

If only 2 sides of the triangle are given, use the Pythagorean Theorem to solve for the missing side

$$a^2 + b^2 = c^2$$

a and b: legs

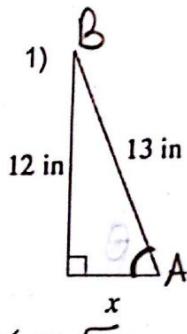
c: hypotenuse

On
Day 3

To solve a triangle means to find the lengths of all the sides and the measures of all the angles.

- o If only 2 sides of the triangle are given, use the Pythagorean Theorem to solve for the third side.
- o If one side and an angle are given, use trig ratios and the calculator to solve for the missing sides.

1)



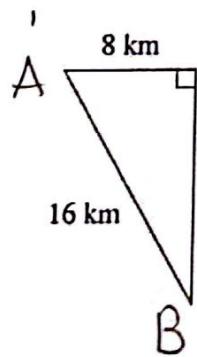
$$x = 5$$

$$\sin A = \frac{12}{13}$$

$$A = 67^\circ$$

$$B = 23^\circ$$

2)



3)

$$a = 11 \text{ m}, c = 15 \text{ m}$$

$$11^2 + b^2 = 15^2$$

$$b = 10.2$$

$$\sin A = \frac{11}{15}$$

$$A = 47^\circ$$

$$B = 43^\circ$$

$$\sin B = \frac{8}{16}$$

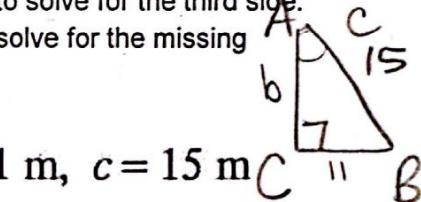
$$B = 30^\circ, A = 60^\circ$$

$$13.9 \sqrt{192}$$

$$8^2 + x^2 = 16^2$$

$$x = 13.9$$

$$\sqrt{104}$$



$$a = 11 \text{ m}, c = 15 \text{ m}$$

$$11^2 + b^2 = 15^2$$

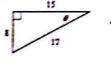
$$b = 10.2$$

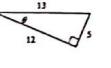
$$\sin A = \frac{11}{15}$$

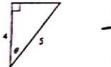
$$A = 47^\circ$$

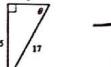
$$B = 43^\circ$$

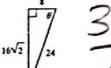
Find the value of the trig function indicated.

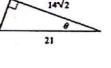
1) $\sec \theta$  $\frac{17}{15}$

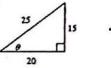
2) $\sec \theta$  $\frac{13}{12}$

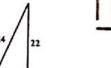
3) $\cot \theta$  $\frac{4}{3}$

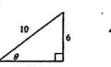
4) $\csc \theta$  $\frac{17}{15}$

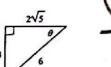
5) $\csc \theta$  $\frac{3\sqrt{2}}{4}$

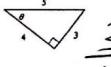
6) $\cos \theta$  $\frac{2\sqrt{2}}{3}$

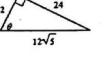
7) $\cot \theta$  $\frac{4}{3}$

8) $\tan \theta$  $\frac{11\sqrt{23}}{23}$

9) $\tan \theta$  $\frac{3}{4}$

10) $\cot \theta$  $\frac{\sqrt{5}}{2}$

11) $\tan \theta$  $\frac{3}{4}$

12) $\cot \theta$  $\frac{1}{2}$

-1-

13) $\tan \theta$  $\frac{7}{24}$



14) $\sin \theta$  $\frac{6}{7}$



Find the value of each. Round your answers to the nearest ten-thousandth.

15) $\cos 10^\circ$ 0.9848

17) $\csc 21^\circ$ 2.7904

19) $\tan 40^\circ$ 0.8391

21) $\csc 56^\circ$ 1.2062

23) $\tan 10^\circ$ 0.1763

Find the value of the trig function indicated.

25) Find $\csc \theta$ if $\tan \theta = \frac{3}{4}$ $\frac{5}{3}$

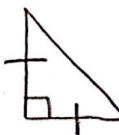
27) Find $\tan \theta$ if $\sin \theta = \frac{4}{5}$ $\frac{4}{3}$

29) Find $\sec \theta$ if $\sin \theta = \frac{3\sqrt{13}}{13}$ $\frac{\sqrt{13}}{2}$

30) Find $\cot \theta$ if $\sin \theta = \frac{12}{13}$ $\frac{5}{12}$

Critical think questions:

- 31) Draw a right triangle that has an angle with a tangent of 1.



Isosceles
Right

- X) What is the slope of the hypotenuse for #9?
How does that compare to $\tan \theta$? Why?

$$\frac{3}{4} = \tan \theta$$

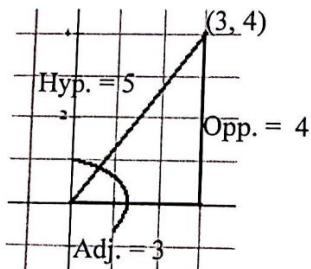
$$\frac{\text{rise}}{\text{run}} = \frac{\text{opp}}{\text{adj}}$$

-2-

Key

Given a point P in the coordinate plane, let P be on the terminal side of an angle θ . Let $P = (3, 4)$.

To draw the triangle:



Use the X-coord as the base of the triangle and the y-coord as the height of the triangle.

Use the pythag to find the length of the hypotenuse.

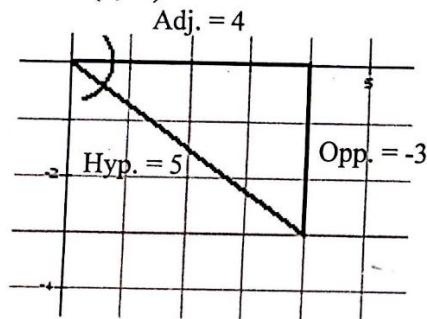
Find the 6 trigonometric ratios of the indicated angle in the resulting triangle.

$$\sin \theta = \frac{4}{5} \quad \cos \theta = \frac{3}{5} \quad \tan \theta = \frac{4}{3}$$

$$\csc \theta = \frac{5}{4} \quad \sec \theta = \frac{5}{3} \quad \cot \theta = \frac{3}{4}$$

EX. A point on the terminal side of θ is given. Find the exact values of the six trigonometric ratios of the indicated angle.

1. $P = (4, -3)$



If one of the coordinates is negative that just means that the triangle will be reflected.

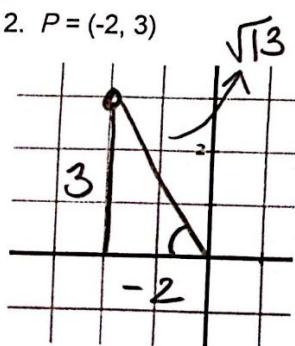
Draw a right triangle using the X-coord as the base of the triangle.

Use the pythag to find the length of the hypotenuse.

Find the 6 trigonometric ratios of the indicated angle in the resulting triangle.

$$\sin \theta = -\frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = -\frac{3}{4} \quad \csc \theta = -\frac{5}{3} \quad \sec \theta = \frac{5}{4} \quad \cot \theta = -\frac{4}{3}$$

2. $P = (-2, 3)$



$$(-2)^2 + 3^2 = c^2$$

$$c = \sqrt{13}$$

$$\sin \theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos \theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$$

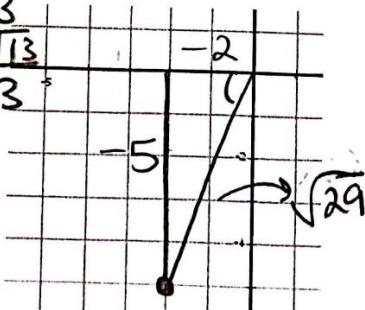
$$\tan \theta = -\frac{3}{2}$$

$$\csc \theta = \frac{\sqrt{13}}{3}$$

$$\sec \theta = \frac{\sqrt{13}}{-2}$$

$$\cot \theta = -\frac{2}{3}$$

3. $P = (-2, -5)$



$$5^2 + 2^2 = c^2$$

$$29 = c^2$$

$$c = \sqrt{29}$$

$$\sin \theta = -\frac{5}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\cos \theta = -\frac{2}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$

$$\tan \theta = \frac{5}{2}$$

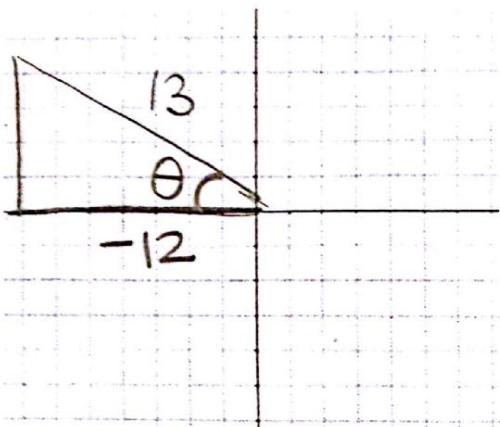
$$\csc \theta = \frac{\sqrt{29}}{5}$$

$$\sec \theta = -\frac{\sqrt{29}}{2}$$

$$\cot \theta = \frac{2}{5}$$

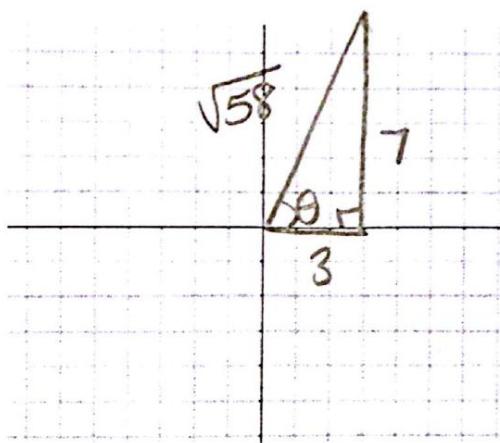
Find all 6 trigonometric ratios of the following by drawing the point in the coordinate plane:

1. (-12, 5)



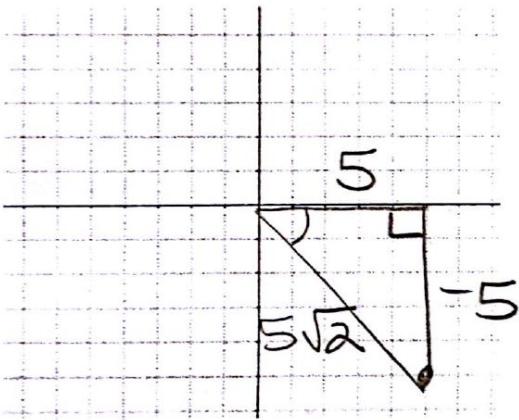
$$\begin{aligned}\sin \theta &= \frac{5}{13} & \cos \theta &= \frac{-12}{13} & \tan \theta &= \frac{-5}{12} \\ \csc \theta &= \frac{13}{5} & \sec \theta &= -\frac{13}{12} & \cot \theta &= -\frac{12}{5}\end{aligned}$$

2. (3, 7)



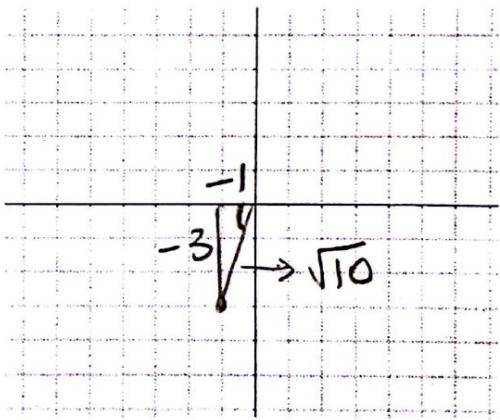
$$\begin{aligned}7^2 + 3^2 &= C^2 & C &= \sqrt{58} \\ 49 + 9 &= C^2 \\ \sin \theta &= \frac{7}{\sqrt{58}} = \frac{7\sqrt{58}}{58} & \cos \theta &= \frac{3}{\sqrt{58}} = \frac{3\sqrt{58}}{58} & \tan \theta &= \frac{7}{3} \\ \csc \theta &= \frac{\sqrt{58}}{7} & \sec \theta &= \frac{\sqrt{58}}{3} & \cot \theta &= \frac{3}{7}\end{aligned}$$

3. (5, -5)



$$\begin{aligned}\sin \theta &= \frac{-5}{5\sqrt{2}} = \frac{-\sqrt{2}}{2} & \cos \theta &= \frac{5}{5\sqrt{2}} = \frac{\sqrt{2}}{2} & \tan \theta &= -1 \\ \csc \theta &= -\frac{1}{\sqrt{2}} & \sec \theta &= \frac{2}{\sqrt{2}} = \sqrt{2} & \cot \theta &= -1\end{aligned}$$

4. (-1, -3)



$$\begin{aligned}\sin \theta &= \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10} & \cos \theta &= \frac{-1}{\sqrt{10}} = \frac{-\sqrt{10}}{10} & \tan \theta &= 3 \\ \csc \theta &= -\frac{\sqrt{10}}{3} & \sec \theta &= -\frac{1}{\sqrt{10}} & \cot \theta &= \frac{1}{3}\end{aligned}$$

$$C = \sqrt{50} = 5\sqrt{2}$$

$$\begin{aligned}3^2 + 1^2 &= C^2 \\ C &= \sqrt{10}\end{aligned}$$