

Released Form

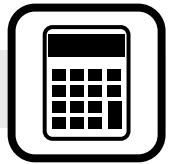
Student Name: _____

Spring 2013
North Carolina
Measures of Student Learning:
NC's Common Exams
**Advanced Functions
and Modeling**



Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314

Student Booklet

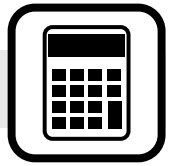


- 1 The table below shows the probability distribution of the number of televisions in each house in a community.

| Televisions | Probability |
|-------------|-------------|
| 0 | 0.04 |
| 1 | 0.38 |
| 2 | 0.27 |
| 3 | x |
| 4 | y |
| 5 or more | 0.13 |

What is the probability that a house in the community will have at least 3 televisions?

- A 0.69
 B 0.31
 C 0.18
 D 0.09
- 2 Anna and Zach each have \$600 to invest. Anna’s investments earn a rate of 10.5%, and Zach’s investments earn a rate of 6.5%. **Approximately**, how much more money will Anna have than Zach when Zach’s investments are worth \$900? (Assume continuous compounding.)
- A \$184
 B \$241
 C \$255
 D \$264



- 3 A solution's pH is given by the function $p(t) = -\log(t)$, where t is the hydronium ion concentration, in moles per liter. A sample of coffee has a pH of 5.0. What is the **approximate** hydronium ion concentration of the sample?

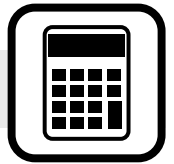
- A 0.000001
- B 0.00001
- C 0.0001
- D 0.001

- 4 A sequence is shown below.

1, 0.1, 0.01, 0.001, 0.0001, . . .

What is the sum of the sequence?

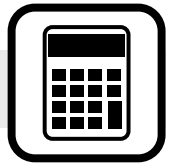
- A $1\frac{1}{10}$
- B $1\frac{1}{9}$
- C $1\frac{2}{9}$
- D $1\frac{9}{10}$



5 Which statement is true about the sequence shown below?

$0, 4.5, 12, 22.5, \dots$

- A The series converges because the limit of the sequence as n approaches infinity is infinity.
- B The series converges because the limit of the sequence as n approaches infinity is 30.
- C The series diverges because the limit of the sequence as n approaches infinity is infinity.
- D The series diverges because the limit of the sequence as n approaches infinity is 30.
- 6 A pharmaceutical company is creating a new cholesterol drug to prevent heart disease. The company must collect data by testing the drug before it is approved. Which would be the **best** method of data collection?
- A experimental study
- B observational study
- C simulation
- D survey



7 The table below shows the midterm and final exam grades of ten students.

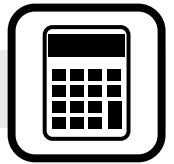
| | | | | | | | | | | |
|-------------------|----|----|----|----|----|----|----|----|----|----|
| Midterm | 68 | 78 | 92 | 90 | 88 | 82 | 94 | 83 | 71 | 62 |
| Final Exam | 62 | 77 | 99 | 87 | 85 | 84 | 95 | 98 | 72 | 64 |

Which comparison between the midterm grades and the final exam grades is true?

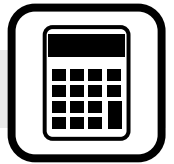
- A The final exam grades have a higher mean and standard deviation than the midterm grades.
- B The final exam grades have a lower mean and standard deviation than the midterm grades.
- C The final exam grades have a higher mean and a lower standard deviation than the midterm grades.
- D The final exam grades have a lower mean and a higher standard deviation than the midterm grades.

8 A baseball team scored the following number of runs in its games this season: 6, 2, 5, 9, 11, 4, 5, 8, 6, 7, 5. There is one more game in the season. If the team wants to end the season with an average of at least 6 runs per game, what is the least number of runs the team must score in the final game of the season?

- A 2
- B 4
- C 6
- D 8



- 9 If the probability of giving birth to a boy is 0.52, what is the **approximate** probability of giving birth to four consecutive boys?
- A 0.021
B 0.062
C 0.073
D 0.130
- 10 How many more ways can 10 juniors running for the positions of president, vice president, secretary, and treasurer be selected when compared to 12 sophomores running for 5 identical positions of class representative?
- A 94,830
B 11,628
C 4,320
D 4,248
- 11 A starting line for a hockey team should consist of 3 offensive players, 2 defensive players, and 1 goaltender. A coach has 11 offensive players, 6 defensive players, and 2 goaltenders from which to choose the starting line. How many unique starting lines can the coach create?
- A 132
B 792
C 4,950
D 59,400



- 12 It costs a bakery \$3.50 to make apple pies that sell for \$12 the first day they are baked.
- If a pie is not sold on the first day, the new price is \$8.50.
 - The probability of selling the apple pie the first day is 75%.
 - There is a 12% probability of selling it on the second day.
 - If the apple pie does not sell by the end of the second day, it is donated.

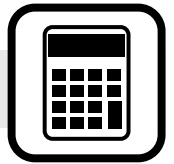
What is the **approximate** expected profit per pie for the bakery on the sale of its apple pies?

- A \$5.67
- B \$6.52
- C \$9.57
- D \$10.02
- 13 The number of household members, x , living in Cityville homes has the following probability distribution:

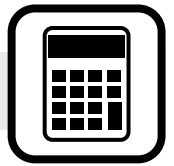
| | | | | | | | | |
|--------|------|------|------|------|------|------|------|------|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $P(x)$ | 0.21 | 0.28 | 0.16 | 0.22 | 0.06 | 0.04 | 0.02 | 0.01 |

What is the expected size of a household in Cityville?

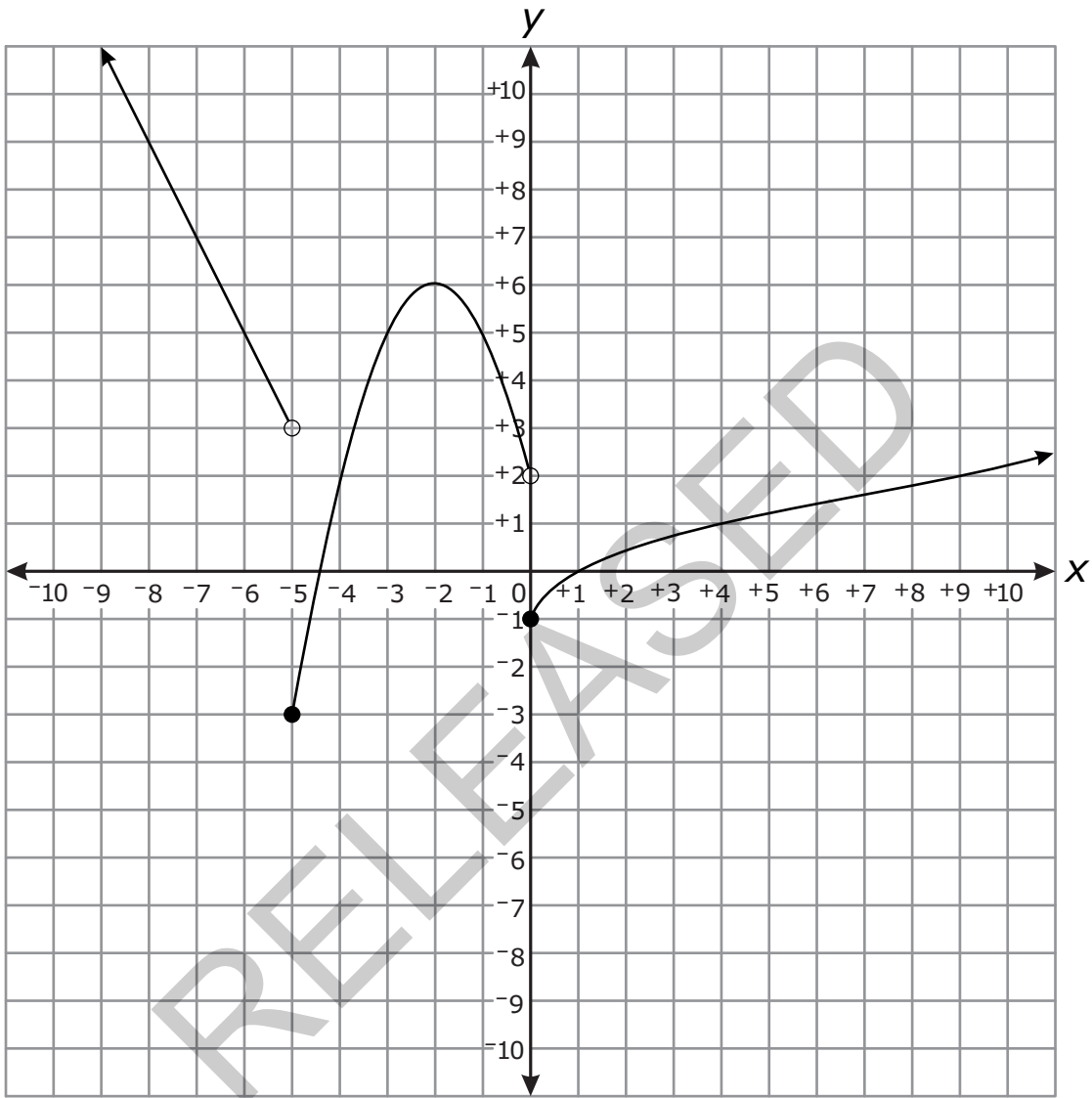
- A 2.43
- B 2.89
- C 3.17
- D 4.50



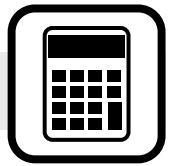
- 14 What is the middle term for the expansion of $(x^2 + 3)^{12}$?
- A $729x^{12}$
B $924x^{12}$
C $673,596x^{12}$
D $665,280x^{12}$
- 15 Abby took an 8-question multiple-choice quiz. Suppose that her probability of correctly answering any question is 0.75. What is Abby's probability of incorrectly answering exactly two questions on the quiz?
- A $P = 0.089$
B $P = 0.240$
C $P = 0.311$
D $P = 0.623$
- 16 Which function results by shifting the graph of $y = \ln(x + 3) - 6$ to the left 4 units and down 3 units?
- A $y = \ln(x + 7) - 9$
B $y = \ln(x - 1) - 9$
C $y = \ln(x + 7) - 3$
D $y = \ln(x - 1) - 3$



17 Which piecewise function is graphed below?



Answer choices are on the following page.



$$A \quad f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x + 2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$$

$$B \quad f(x) = \begin{cases} -2x - 7 & \text{for } x < -5 \\ -(x - 2)^2 + 6 & \text{for } -5 \leq x < 0 \\ \sqrt{x} - 1 & \text{for } x \geq 0 \end{cases}$$

$$C \quad f(x) = \begin{cases} -2x - 7 & \text{for } x \leq -5 \\ -(x - 2)^2 + 6 & \text{for } -5 < x \leq 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$$

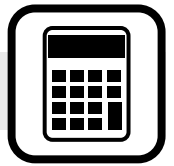
$$D \quad f(x) = \begin{cases} -2x - 7 & \text{for } x \leq -5 \\ -(x + 2)^2 + 6 & \text{for } -5 < x \leq 0 \\ \sqrt{x} - 1 & \text{for } x > 0 \end{cases}$$

18 A function, $f(x)$, is shown below.

$$f(x) = \begin{cases} x - 4 & \text{for } 0 \leq x < 2 \\ x^2 - 3x + 4 & \text{for } 2 \leq x < 4 \\ 5 & \text{for } 4 \leq x < 7 \end{cases}$$

What is the range of $f(x)$?

- A $[-4, 5)$
- B $[-4, 8)$
- C $[-4, -2) \cup [2, 5)$
- D $[-4, -2) \cup [2, 8)$



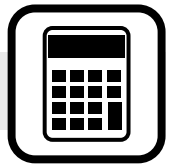
- 19 A water tower is located 410 feet from a building. From a window in the building, it is observed that the angle of elevation to the top of the tower is 42 degrees and the angle of depression to the bottom of the tower is 25 degrees. **Approximately** how tall is the water tower?
- A 191 feet
 - B 369 feet
 - C 448 feet
 - D 560 feet

20 Given the table below:

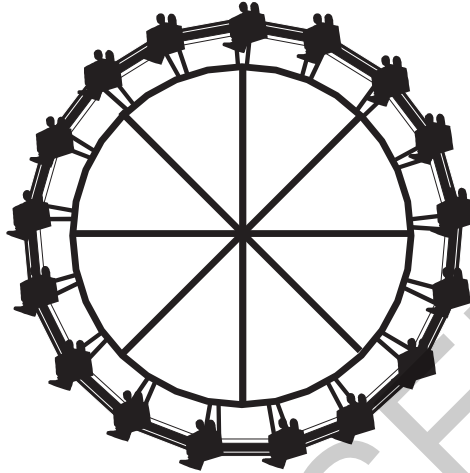
| | | | | | |
|----------|-----------------|------------------|-------|------------------|------------------|
| x | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | π | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ |
| y | 0.5 | 0 | -0.5 | 0 | 0.5 |

Which function fits the data?

- A $y = 0.5 \cos(2x - \pi)$
- B $y = 0.5 \cos(x - \pi)$
- C $y = 0.5 \cos\left(2x + \frac{\pi}{2}\right)$
- D $y = \cos\left(2x + \frac{\pi}{2}\right)$

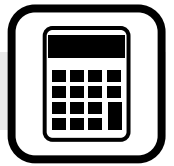


- 21 A Ferris wheel has a diameter of 80 feet. Riders enter the Ferris wheel at its lowest point, 5 feet above the ground, at time $t = 0$ seconds. One complete rotation takes 65 seconds.

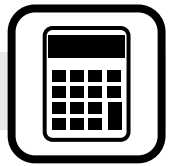


Which function models a rider's vertical height, $h(t)$, at t seconds?

- A $h(t) = -80 \cos\left(\frac{2\pi}{65}t\right) + 5$
- B $h(t) = -40 \cos\left(\frac{2\pi}{65}t\right) + 45$
- C $h(t) = -45 \cos\left(\frac{65}{2\pi}t\right) + 40$
- D $h(t) = -5 \cos\left(\frac{65}{2\pi}t\right) + 80$



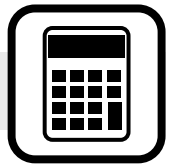
- 22 How does the graph of $g(x) = 0.5\cos(2x)$ differ from the graph of its parent function, $f(x) = \cos(x)$, over the interval $-\pi \leq x \leq \pi$?
- A The amplitude is smaller, and the period is shorter.
 - B The amplitude is smaller, and the period is longer.
 - C The amplitude is larger, and the period is shorter.
 - D The amplitude is larger, and the period is longer.
- 23 Two sides of a triangle measure 14 ft and 17 ft, respectively. The included angle is 72° . **Approximately** how long is the third side of the triangle?
- A 18.4 ft
 - B 20.3 ft
 - C 25.1 ft
 - D 30.7 ft
- 24 In a geometric sequence, $a_1 = 12$ and $r = \sqrt{2}$. What is the **approximate** sum of the first 20 terms of the sequence?
- A 339.4
 - B 8,688.9
 - C 29,624.9
 - D 29,636.9



- 25 A bathroom floor has tiles arranged in 9 circles. The innermost circle contains 9 tiles. Each successive circle contains 9 more tiles than the previous circle. How many total tiles are on the bathroom floor?
- A 81
 - B 396
 - C 405
 - D 729

This is the end of the multiple-choice portion of the test.

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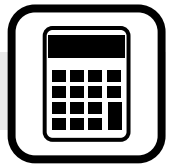
The questions you read next will require you to answer in writing.

1. Write your answers on separate paper.
2. Be sure to write your name on each page.

1 The table below shows the estimated average hours each person in a city spent playing video games in different years.

| Years since 2002 | Hours |
|------------------|-------|
| 0 | 71 |
| 1 | 80 |
| 2 | 82 |
| 3 | 78 |
| 4 | 80 |
| 5 | 91 |
| 6 | 107 |
| 7 | 121 |
| 8 | 125 |
| 9 | 131 |
| 10 | 142 |

- Write an equation for the best fit exponential model for the data.
- What is the meaning of the base of the model in the context of the problem?
- What is the meaning of the coefficient of the model in the context of the problem?



- 2 Students are told that attending class regularly will help improve their scores in that class. Below are the scores for students who did attend class regularly and scores for those who did not.

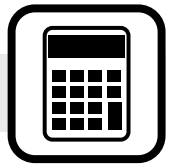
| Attended Class Regularly | | | | | Did Not Attend Class Regularly | | | |
|--------------------------|-----|-----|-----|-----|--------------------------------|-----|-----|-----|
| 241 | 261 | 271 | 282 | 296 | 185 | 195 | 195 | 228 |
| 243 | 262 | 272 | 284 | 296 | 250 | 256 | 225 | 261 |
| 254 | 267 | 278 | 292 | 308 | 274 | 277 | 308 | 233 |
| 252 | 264 | 276 | 290 | 310 | | | | |

- Which group of students has a larger mean score and by how much?
- Which group of students has a larger median score and by how much?

- 3 Write an equation for the power function, in $y = ax^b$ form, that passes through the points (2, 1) and (5, 6).

- Use your power function to predict the value of y when $x = 9$.

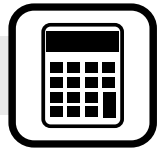
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This is the end of the Advanced Functions and Modeling test.

- 1. Look back over your answers.**
- 2. Put all of your papers inside your test book and close the test book.**
- 3. Place your calculator on top of the test book.**
- 4. Stay quietly in your seat until your teacher tells you that testing is finished.**

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Advanced Functions and Modeling
RELEASED Form – Paper/Pencil Version
Spring 2013
Answer Key

| Item number | Type | Key | Competency Goal(s) |
|-------------|------|-----|-----------------------------------|
| 1 | MC | B | 1 — Data Analysis and Probability |
| 2 | MC | C | 2 — Algebra |
| 3 | MC | B | 2 — Algebra |
| 4 | MC | B | 2 — Algebra |
| 5 | MC | C | 2 — Algebra |
| 6 | MC | A | 1 — Data Analysis and Probability |
| 7 | MC | A | 1 — Data Analysis and Probability |
| 8 | MC | B | 1 — Data Analysis and Probability |
| 9 | MC | C | 1 — Data Analysis and Probability |
| 10 | MC | D | 1 — Data Analysis and Probability |
| 11 | MC | C | 1 — Data Analysis and Probability |
| 12 | MC | B | 1 — Data Analysis and Probability |
| 13 | MC | B | 1 — Data Analysis and Probability |
| 14 | MC | C | 1 — Data Analysis and Probability |
| 15 | MC | C | 1 — Data Analysis and Probability |
| 16 | MC | A | 2 — Algebra |
| 17 | MC | A | 2 — Algebra |
| 18 | MC | D | 2 — Algebra |
| 19 | MC | D | 2 — Algebra |
| 20 | MC | A | 2 — Algebra |
| 21 | MC | B | 2 — Algebra |
| 22 | MC | A | 2 — Algebra |
| 23 | MC | A | 2 — Algebra |



| Item number | Type | Key | Competency Goal(s) |
|-------------|------|--------|-----------------------------------|
| 24 | MC | D | 2 — Algebra |
| 25 | MC | C | 2 — Algebra |
| 26 | CR | Rubric | 1 — Data Analysis and Probability |
| 27 | CR | Rubric | 1 — Data Analysis and Probability |
| 28 | CR | Rubric | 2 — Algebra |

Item Types:

MC = multiple choice

CR = constructed response

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