The value of tolls on the New Jersey Turnpike is based upon the number of miles traveled. The function $t(x) = \begin{cases} 3x+15 & 0 \le x \le 95 \\ 11x - 725 & 95 < x \le 112 \end{cases}$ can be used to calculate the cost of traveling along the NJ Turnpike. Where x represents the number of miles traveled (rounded to the nearest mile) and t(x) represents the cost of the toll in cents.

1) What is the domain of the piecewise function in context? (Explain what it means) x = # miles

2) State the overall domain for the entire function. [0, 112]

3) Find the cost of traveling the entire length of the NJ Turnpike (112 miles). 11(112) − 725 = 507¢ = $5.07

4) About how many miles would you need to travel to have a toll of $2.50?

95 miles 3(95)+15 = $3.00

3x+15 = 250

x ≈ 78 miles

You have a summer job that pays time and a half for overtime. That is, if you work more than 40 hours per week, your hourly wage for the extra hours is 1.5 times your normal hourly wage of $7. The piecewise function is:

$$P(h) = \begin{cases} 7h, & \text{if } 0 \le h \le 40 \\ 10.5h - 140, & \text{if } h > 40 \end{cases}$$

1) How much would you get paid for working a job Monday through Friday 9 am - 5 pm? 40 hrs 7(40) = $280

2) How much would you get paid if you worked 45 hours?

10.5(45)−140 =

$332.50

(1)

APPLICATIONS OF PIECEWISE FUNCTIONS

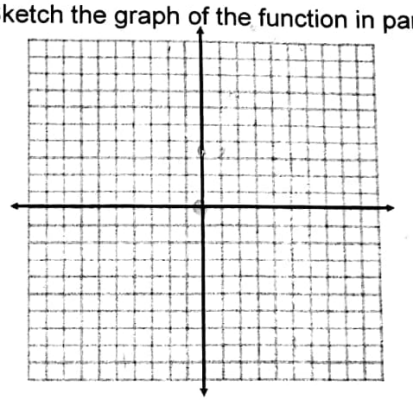
Key

Example 1: ***Admission fees.*** A local zoo charges admission to groups according to the following policy. Groups of fewer than 50 people are charged a rate of $35.00 per person, while groups of 50 people or more are charged a reduced rate of $30.00 per person.

(a) Find a mathematical model expressing the amount a group will be charged for admission as a function of its size.

$$f(x) = \begin{cases} 35x & 1 \le x < 50 \\ 30x & x \ge 50 \end{cases}$$

(b) Sketch the graph of the function in part a)



(c) How much money will a group of 49 people save in admission cost if it can recruit one additional member?

$1715 - 1500 = \$215$

(4)

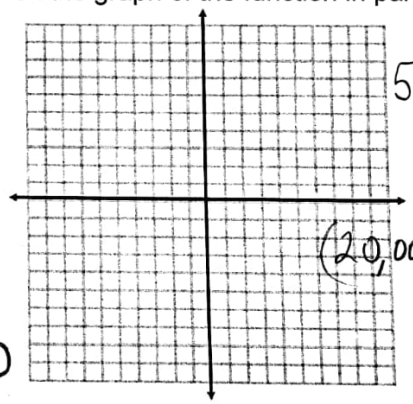
Example 2: ***Income tax rates.*** A certain country taxes the first $20 000 of an individual's income at a rate of 15% and all income over $20 000 is taxed at 20%.

(a) Find a piecewise-defined function T that gives the total tax on an income of x dollars.

$$f(x) = \begin{cases} .15x & 0 < x \le 20{,}000 \\ .2(x - 20{,}000) + 3000 & x > 20{,}000 \end{cases}$$

$.15(20{,}000) = 3000$

(b) Sketch the graph of the function in part (a).



ex) 50,000

$(20{,}000).15 = 3000$

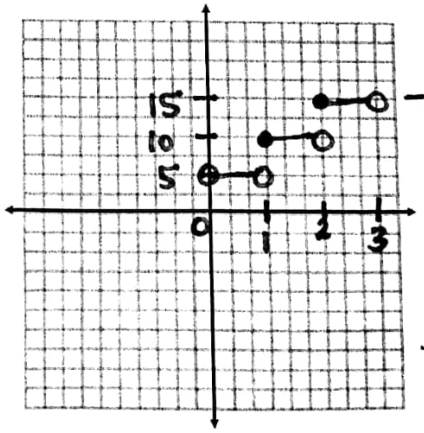
(2)

Example 3: ***Parking Costs.*** A parking garage charges 5.00 for up to (but not including) 1 hour of parking, 10.00 for up to 2 hours parking, 15.00 for up to 3 hours parking and so on.

(a) Find a piecewise-defined function C that gives the cost of parking for t hours.

$$f(x) = \begin{cases} 5.00 & 0 < x < 1 \\ 10.00 & 1 \le x < 2 \\ 15.00 & 2 \le x < 3 \end{cases}$$

(b) Sketch the graph of the function in part (a).



50,000
−20,000
30,000

$.2(30{,}000) = 6000$
$+3000$
9000

(c) How much will a person pay if he has parked his car for 4.5 hours?

$20.00 \quad 3 \le x < 4$

$25.00 \quad 4 \le x < 5$

\$25

4 3

Example 4: ***Income Tax in North Carolina!*** The following chart shows the income tax rates in North Carolina for single people.

ex) 50,000 salary

765 + .07(37250)

3372.50

If your filing status is single:

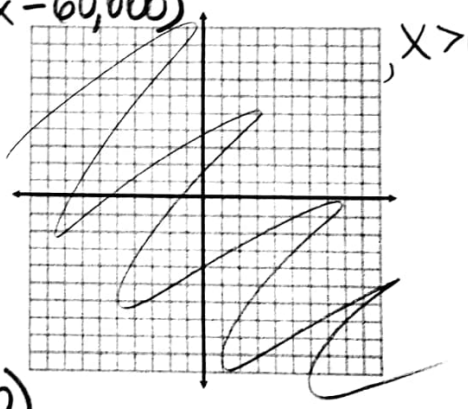
and taxable income is more than:	but not over:	your tax is:
$ 0	$12,750	6% OF THE NC TAXABLE INCOME AMOUNT ON FORM D-400
$12,750	$60,000	$765 – 7% OF THE AMOUNT OVER $12,750
$60,000	____	$4,072.50 – 7.75% OF THE AMOUNT OVER $60,000

12750(.06) = 765

(a) Find a piecewise defined function T that gives the total tax on an income of x dollars.

$$f(x) = \begin{cases} .06x, & 0 < x \le 12750 \\ 765 + .07(x - 12750), & 12750 < x \le 60000 \\ 4072.50 + .0775(x - 60{,}000), & x > 60000 \end{cases}$$

(b) Sketch the graph of the function in part (a).



(c) Suppose your yearly salary is $30,000 dollars.

a. How much money will you pay in income tax?

765 + .07(30000 − 12750)

$1972.50

b. What will your net income be *after you pay your taxes*?

30000 − 1972.50 = $28,027.50

domain
(0, ∞)
(0, ∞)

(d) Would you rather make a salary of $58,000 a year or $61,000 a year? ***WHY?***

765 + .07(58000 − 12750) → 4072.50 + .0775(61000 − 60000)

3932.50

54067.50
net

4150

56850
net