

Released Items

Student Name: _____

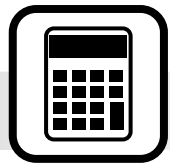
Fall 2015
NC Final Exam
**Advanced Functions and
Modeling**



Student Booklet



Public Schools of North Carolina
State Board of Education
Department of Public Instruction
Raleigh, North Carolina 27699-6314



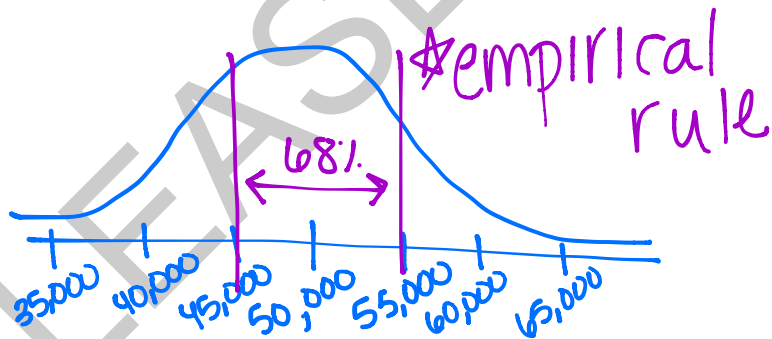
- 1 Suppose the function $H(t) = 8.5\sin(0.017t - 1.35) + 12$ models the hours of sunlight for a town in Alaska, where $t = 1$ is the first day of the year. Based on the function, what is the **approximate** range of daylight hours for the town?

- A 3.5 to 20.5
- B 4 to 20
- C 4.5 to 19.5
- D 5 to 19

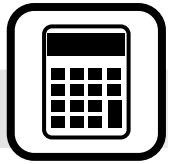
midline @ $y=12$
 amplitude = 8.5
 $12 \pm 8.5 = [3.5, 20.5]$

- 2 The lifetime of a particular type of car tire is normally distributed. The mean lifetime is 50,000 miles, with a standard deviation of 5,000 miles. Of a random sample of 15,000 tires, how many of the tires are expected to last for between 45,000 and 55,000 miles?

- A 7,125
- B 10,200
- C 14,250
- D 14,850



$15,000(0.68) = 10,200$



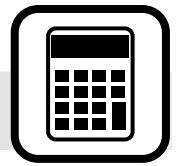
- 3 The frequency table below shows the number of runners in specific age groups for a certain race.

Age Group	Number of Runners
0-10	
11-20	
21-30	
31-40	
41-50	
51-60	
61-70	
71-80	
81-90	

tail on right

What is the shape of the distribution?

- A uniform
- B skewed right**
- C skewed left
- D normal



- 4 A spinner labeled 1 to 9 gives each of the numbers 2, 5, 7, and 9 a $\frac{3}{20}$ chance of being landed upon. The chance of landing on each of the other five numbers is equal. If the spinner is spun 1,000 times, which choice is the **most likely** outcome for the 1,000 spins?

about 150 times

$$\frac{3}{20} (1000) = 150$$

A

Number on Spinner	1	2	3	4	5	6	7	8	9
Number of Occurrences	110	112	111	111	109	112	112	111	112

B

Number on Spinner	1	2	3	4	5	6	7	8	9
Number of Occurrences	82	148	78	80	149	79	151	81	152

about 150

C

Number on Spinner	1	2	3	4	5	6	7	8	9
Number of Occurrences	120	122	100	103	108	126	113	104	104

D

Number on Spinner	1	2	3	4	5	6	7	8	9
Number of Occurrences	121	100	119	120	102	120	98	121	99



5 A group of 12 people need to form a line. The line will consist of exactly 9 of the people. Person X and Person Y have to be either third or fourth in line. How many different orders are possible?

A 79,833,600

B 1,209,600

C 604,800

D 362,880

$$\underline{10} \quad \underline{9} \quad \frac{2}{\begin{matrix} X \\ Y \end{matrix}} \quad \frac{1}{\begin{matrix} X \\ Y \end{matrix}} \quad \underline{8} \quad \underline{7} \quad \underline{6} \quad \underline{5} \quad \underline{4}$$

6 The probability that it will rain on Saturday is $\frac{2}{3}$. The probability that the temperature on Saturday will reach 100°F is $\frac{4}{9}$. The probability that it will rain or reach 100°F on Saturday is $\frac{4}{5}$. What is the probability it will rain and reach 100°F on Saturday?

A $\frac{14}{45}$

B $\frac{16}{45}$

C $\frac{24}{45}$

D $\frac{26}{45}$

$$P(R) = \frac{2}{3}$$

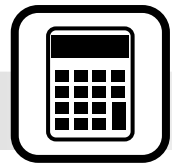
$$P(100) = \frac{4}{9}$$

$$P(R \text{ or } 100) = \frac{4}{5}$$

$$P(R \text{ and } 100) = ?$$

$$P(R \text{ or } 100) = P(R) + P(100) - P(R \text{ and } 100)$$

$$\begin{aligned} \frac{4}{5} &= \frac{2}{3} + \frac{4}{9} - X \\ \frac{4}{5} &= \frac{6}{9} + \frac{4}{9} - X \\ \frac{4}{5} &= \frac{10}{9} - X \\ \frac{4}{5} - \frac{10}{9} &= -X \\ \frac{36}{45} - \frac{50}{45} &= -X \\ \frac{+14}{45} &= +X \end{aligned}$$



- 7 A manufacturing plant produces a special kind of lightbulb.
- Each lightbulb produced has a 0.040 probability of being defective.
 - Five lightbulbs are chosen at random from the production line.

To the nearest thousandth, what is the probability that exactly two of the five bulbs will be defective?

- A 0.014
- B 0.016
- C 0.018
- D 0.020

in calc:

2nd VARS → binompdf

trials: 5

P: 0.04

X-value: 2

binompdf

OR binompdf(5, 0.04, 2)

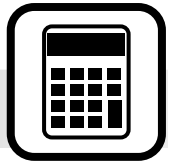
- 8 What is the meaning of the base of the function $y = -\log(x)$?

- A As y decreases by 1, x increases by a factor of 10.
- B As y decreases by 1, x increases by 10.
- C As y increases by 1, x increases by a factor of 10.
- D As y increases by 1, x increases by 10.

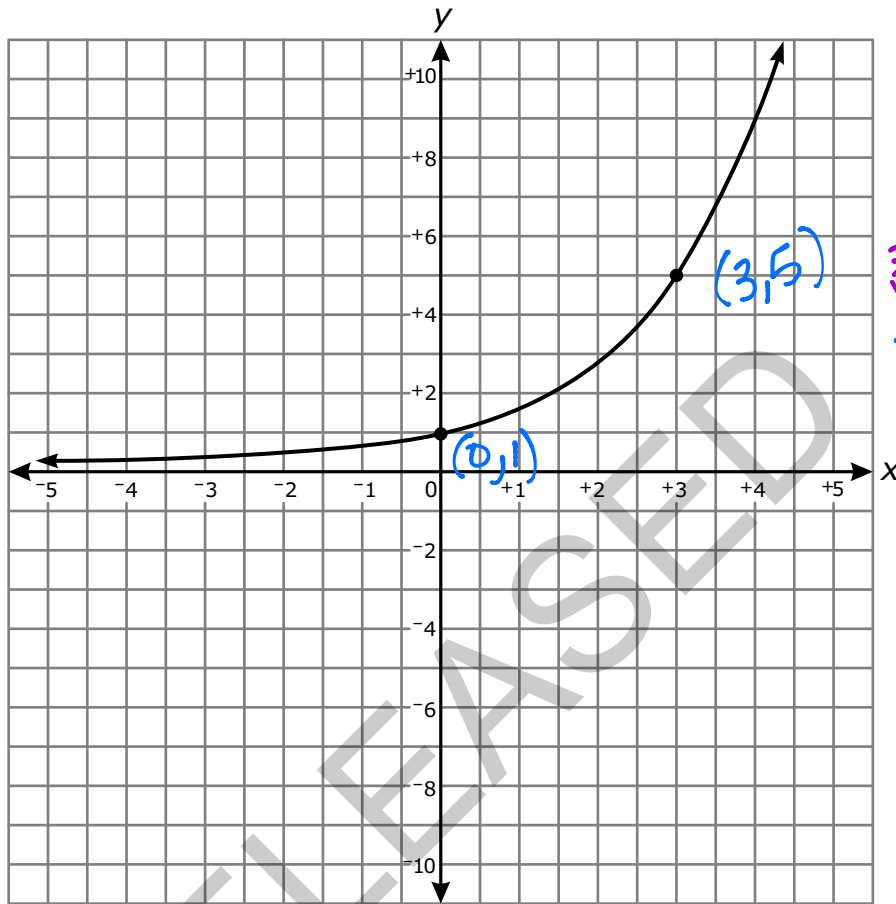
$$y = -\log(x)$$

$$-y = \log(x)$$

$$10^{-y} = x$$



9 The graph of $y = a^x$ is shown below.



$y = a^x$
 $1 = a^0$
 $\sqrt[3]{5} = \sqrt[3]{a^3}$
 $\sqrt[3]{5} = a$

Which choice is closest to $\log_a 3$?

- A 0.9
- B 2.1**
- C 3.2
- D 4.8

$\log_{\sqrt[3]{5}} 3$
 OR
 $\frac{\log 3}{\log \sqrt[3]{5}}$

$= 2.05$



10 A piecewise function is shown below.

$$h(x) = \begin{cases} -2x^2 + 5x + 10 & \text{for } -4 \leq x < 3 \\ 2x + 3p & \text{for } 3 \leq x \leq 5 \end{cases}$$

For what value of p will the function be continuous?

A $\frac{10}{3}$

B $\frac{1}{3}$

C $-\frac{25}{3}$

D $-\frac{34}{3}$

Handwritten work for question 10:

$$-2(3)^2 + 5(3) + 10 = 2(3) + 3p$$

$$-2(9) + 15 + 10 = 6 + 3p$$

$$-18 + 25 = 6 + 3p$$

$$7 = 6 + 3p$$

$$-6 \quad -6$$

$$\frac{1}{3} = \frac{3p}{3} \quad p = \frac{1}{3}$$

11 The equation $y = 4.7x^{\frac{1}{6}}$ is graphed on the coordinate plane. How does increasing the denominator of the exponent transform the graph?

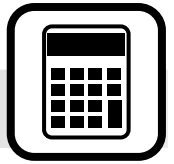
A The transformed graph will approach a horizontal asymptote while the original graph will not.

B The transformed graph will not approach a horizontal asymptote while the original graph will.

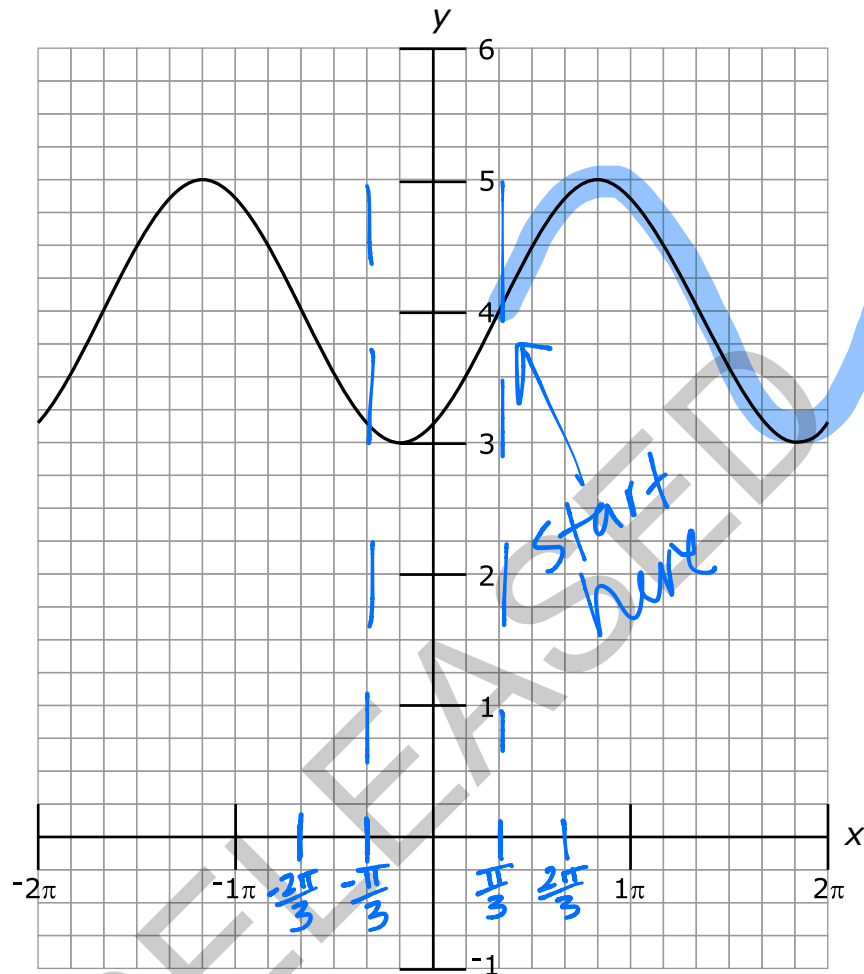
C The transformed graph will go to ∞ slower than the original graph as the value of x gets larger.

D The transformed graph will go to ∞ faster than the original graph as the value of x gets larger.

☆ graph different examples ☆



12 Which function correctly represents the graph below?



A $y = \sin\left(x - \frac{\pi}{3}\right) + 4$

B $y = \sin\left(x + \frac{\pi}{3}\right) + 4$

C $y = \cos\left(x - \frac{\pi}{3}\right) + 4$

D $y = \cos\left(x + \frac{\pi}{3}\right) + 4$

Shifts right
 $\frac{\pi}{3}$



- 13 Which function has an amplitude that is twice the size and a period that is three times the size of the function $y = 3 \cos\left(\frac{x}{4} - 1\right) + 2$?

A $y = 6 \sin\left(\frac{x}{12} - 3\right) + 1$

~~B~~ $y = \frac{3}{2} \cos\left(\frac{3x}{4} + 1\right) - 3$

~~C~~ $y = 6 \cos\left(\frac{3x}{4} - 1\right) + 3$

~~D~~ $y = \frac{3}{2} \sin\left(\frac{x}{12} + 3\right) - 1$

Handwritten notes for question 13:
 $x \times 2 = 6$
 $B = \frac{1}{4}$
 $Per = \frac{2\pi}{1/4} = 8\pi$
 $\times 3 = 24\pi$
 $B = \frac{2\pi}{24\pi} = \frac{1}{12}$

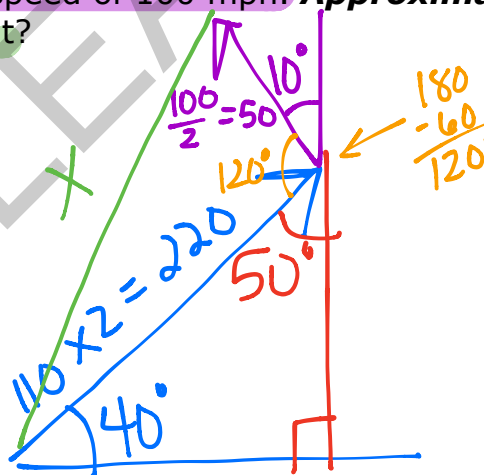
- 14 A plane takes off and travels at an angle of 40° north of east at 110 mph for 2 hours. It then adjusts its path to head 10° west of north and travels in that direction for half an hour at a speed of 100 mph. **Approximately** how far away is the plane from its starting point?

A 182 miles

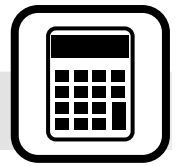
B 200 miles

C 238 miles

D 249 miles



Handwritten solution for question 14:
 $a^2 = b^2 + c^2 - 2(b)(c)\cos A$
 $x^2 = 220^2 + 50^2 - 2(220)(50)\cos(120)$
 $x^2 = 61900$
 $x = 248.797$



15 Which statement is true about the fifth terms of the two sequences below?

explicit $a_n = 3n^2 - 6$ $a_5 = 3(5)^2 - 6 = 69$
recursive $b_n = 3(b_{n-1} - 6); b_1 = 10, 12, 18, 36, 90$
 b_5

- A The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 63.
- B The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 63.
- C The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 21.
- D The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 21.

16 Which statement is true about the series shown below?

$$-4 + -2 + -1 + \frac{-1}{2} + \frac{-1}{4} + \dots$$

- A The series converges because $|r| < 1$.
- B The series diverges because $|r| < 1$.
- C The series converges because $|r| > 1$.
- D The series diverges because $|r| > 1$.

$r = \frac{1}{2}$
 converges



17 What is the explicit form of the equation $a_n = a_{n-1} + 2(n - 1)$; $a_1 = 1$?

A $a_n = 2n - 1$

B $a_n = n^2 - n + 1$

C $a_n = n^2 - 2n + 2$

D $a_n = 2n^2 - 2n - 1$

n	a_n
0	
1	1
2	3
3	7
4	13
5	21

enter in y=

18 An investor bought 1,500 shares of a stock for \$6 a share. He estimates the probability that the stock will rise to a value of \$25 a share is 24%, and the probability it will fall to \$2 a share is 76%. What is the expected value of the investor's profit from buying the stock?

A \$13,560

B \$9,120

C \$6,720

D \$2,280

$19(0.24) - 4(0.76)$

$25 - 6$

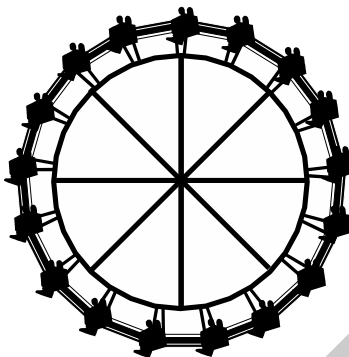
$6 - 2$

RELEASED



- 19 A Ferris wheel is designed in such a way that the height (h), in feet, of the seat above the ground at any time, t , is modeled by the function

$$h(t) = 60 - 55 \sin\left(\frac{\pi}{10}t + \frac{\pi}{2}\right).$$



What is the **maximum** height a seat reaches?

- A 55 feet
- B 60 feet
- C 110 feet
- D 115 feet**

Handwritten work for Question 19:

midline @ $y = 60$
 amplitude = 55
 $\text{max} = 60 + 55 = 115$

- 20 A teacher counts the final exam as 25% of each student's class grade. The remaining 75% is the mean of the student's test scores from the semester. Jaleesa's test scores for the semester are 86, 90, 92, and 80. What is the **minimum** score Jaleesa must get on the final exam to have a class grade of 85.0 or higher?

- A 77
- B 79
- C 81
- D 83

Handwritten work for Question 20 (Step 1):

$$\frac{86 + 90 + 92 + 80}{4} = 87$$

Handwritten work for Question 20 (Step 2):

$$87(0.75) + X(0.25) = 85$$

Handwritten work for Question 20 (Step 3):

$$65.25 + X(0.25) = 85$$

$$-65.25 \quad -65.25$$

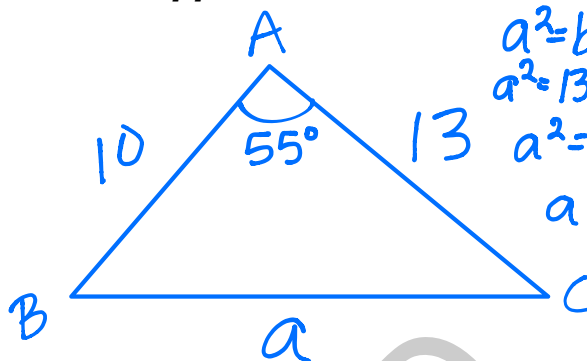
Handwritten work for Question 20 (Step 4):

$$\begin{array}{r} X(0.25) = 19.75 \\ \hline 0.25 \quad 0.25 \\ \hline X = 79 \end{array}$$



- 21 Two sides of a triangle measure 10 inches and 13 inches. The included angle between these sides is 55° . What is the **approximate** measure of the third side of the triangle?

- A 10.9 inches
- B 11.2 inches
- C 13.9 inches
- D 16.2 inches



$$a^2 = b^2 + c^2 - 2(b)(c) \cos A$$

$$a^2 = 13^2 + 10^2 - 2(13)(10) \cos 55$$

$$a^2 = 119.87$$

$$a = 10.95$$

- 22 The third term of a geometric sequence is 96, and the fifth term is 1,536. What is the sum of the first ten terms of this sequence?

- A 4,092
- B 1,572,864
- C 2,097,150
- D 33,554,400

$$a_3 = 96$$

$$a_5 = 1536$$

$$a_3 = a_1 \cdot r^2$$

$$96 = a_1 \cdot r^2$$

$$a_5 = a_1 \cdot r^4$$

$$1536 = a_1 \cdot r^4$$

$$\frac{1536}{96} = \frac{a_1 \cdot r^4}{a_1 \cdot r^2}$$

$$16 = r^2$$

$$r = 4$$

$$S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_{10} = 6 \left(\frac{1-4^{10}}{1-4} \right)$$



1 What transformations have occurred to create the function $f(x) = 3x^3 - 4$ from the function $g(x) = x^3$?

- A The graph of the function has been stretched horizontally and shifted up four units.
- B The graph of the function has been stretched vertically and shifted up four units.
- C The graph of the function has been stretched horizontally and shifted down four units.
- D The graph of the function has been stretched vertically and shifted down four units.

2 An object is launched straight upward from ground level with an initial velocity of 50.0 feet per second. The height, h (in feet above ground level), of the object t seconds after the launch is given by the function $h(t) = -16t^2 + 50t$. At **approximately** what value of t will the object have a height of 28.0 feet and be traveling downward?

- A 2.39 seconds
- B 1.84 seconds
- C 1.56 seconds
- D 0.73 seconds

$$28 = -16t^2 + 50t$$

$$-28 \qquad \qquad \qquad -28$$

$$0 = -16t^2 + 50t - 28$$

★graph and find x-intercepts★

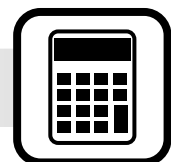
$$t = .73, t = 2.39$$

graph original function

3 What is the range of the function $f(x) = -5 - 2(x + 3)^2$?

- A $[-5, \infty)$
- B $(-\infty, 5]$
- C $(-\infty, -5]$
- D $(-\infty, \infty)$

★graph in calc★



omit

A wind that is blowing from the northwest toward the southeast can be represented by a vector. The vector has an eastward component and a southward component. If the eastward component has a magnitude of 5.00 miles per hour and the southward component has a magnitude of 15.00 miles per hour, in what direction is the wind blowing?

- A The wind is blowing in the direction 71.6° east of south.
- B The wind is blowing in the direction 67.5° east of south.
- C The wind is blowing in the direction 22.5° east of south.
- D The wind is blowing in the direction 18.4° east of south.

5 What value of x satisfies the equation $\log_3(x - 4) = 2$?

- A 5
- B 10
- C 12
- D 13**

$$3^2 = x - 4$$

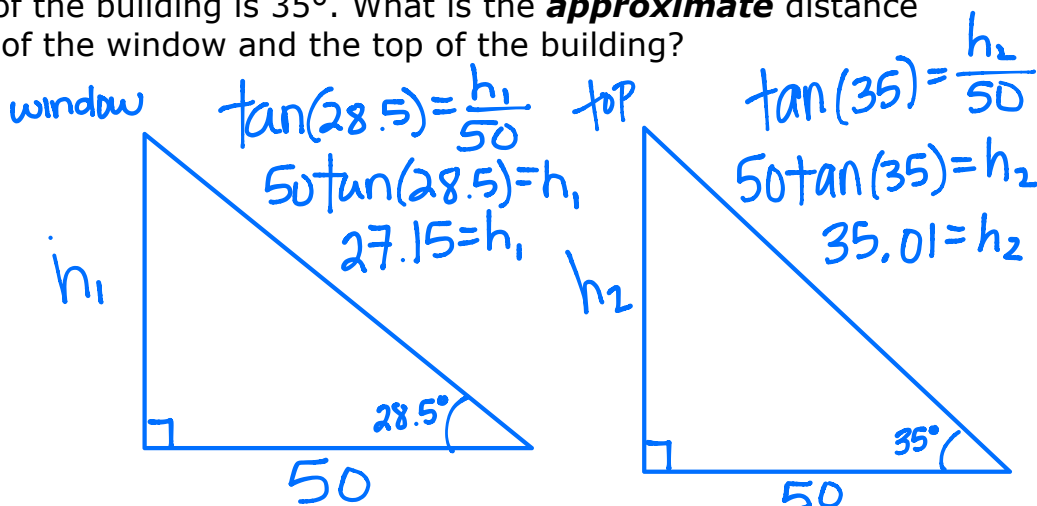
$$9 = x - 4$$

$$+4 \quad +4$$

$$13 = x$$

6 A man is standing on level ground 50 feet away from the wall of a building. He looks up at a window on the building. The angle of elevation to the bottom of the window is 28.5° . He then looks up at the top of the building. The angle of elevation to the top of the building is 35° . What is the **approximate** distance between the bottom of the window and the top of the building?

- A 5.7 feet
- B 7.9 feet**
- C 8.3 feet
- D 8.5 feet



$$h_2 - h_1 = 7.86$$



7
omit

Triangle WXY has the following properties:

- The angle at vertex W is 14° , and the angle at vertex X is obtuse.
- The side opposite vertex W has a length of 7.00 units.
- The side opposite vertex X has a length of 9.00 units.

What is the **approximate** length of the side opposite vertex Y ?

- A 1.73 units
- B 2.08 units
- C 3.26 units
- D 5.40 units

8 Consider these two trigonometric functions:

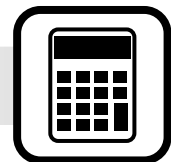
$$f(x) = 3 \sin(2x) + 4$$

$$g(x) = 3 \sin\left(2x - \frac{\pi}{2}\right) + 4$$

$2x - \frac{\pi}{2} = 0$
 $+\frac{\pi}{2}$
 $\frac{1}{2} \cdot 2x = \frac{\pi}{2} \cdot \frac{1}{2}$
 $x = \frac{\pi}{4}$

How should the graph of f be shifted to produce the graph of g ?

- A Shift the graph of f to the left $\frac{\pi}{4}$ units to produce the graph of g .
- B Shift the graph of f to the right $\frac{\pi}{4}$ units to produce the graph of g .**
- C Shift the graph of f to the left $\frac{\pi}{2}$ units to produce the graph of g .
- D Shift the graph of f to the right $\frac{\pi}{2}$ units to produce the graph of g .



~~13~~
omit

Two parametric equations are shown below, where $t \geq 0$.

$$x = \frac{1}{3}\sqrt{t} + 3$$

$$y = 4t^2 - 7$$

Which nonparametric equation can be used to graph the curve described by the parametric equations?

A $y = \frac{4}{9}(x + 1) - 7$

B $y = \frac{4}{3}(x + 3) - 7$

C $y = 36(x - 1)^4 - 7$

D $y = 324(x - 3)^4 - 7$

14 The formula for a sequence is shown below.

$$a_n = 2a_{n-1} + 3, a_1 = 3$$

Which is another formula that represents the sequence?

A $f(n) = 3(2^n - 1)$

B $f(n) = 2n^3 - 3n^2 + 8n + 3$

C $f(n) = 2(n^2 + 1)$

D $f(n) = 3n^2 + 8n - 1$

put
in
y=

n	a _n
1	3
2	9
3	21
4	45
5	93



15 When $a_1 = 25,000$, what is the sum of the infinite sequence defined by the equation $a_{n+1} = 0.8a_n$?

- A 125,000
- B 140,000
- C 160,000
- D 195,000

n	a_n
1	25,000
2	20,000
3	16,000
4	12,800
5	10,240

$\downarrow \times 0.8$
 $\downarrow \times 0.8$
 $\downarrow \times 0.8$
 $\downarrow \times 0.8$

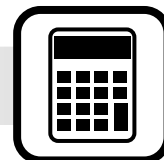
$r = 0.8$
 converges
 $S = \frac{a_1}{1-r} = \frac{25000}{1-0.8}$
 $S = 125,000$

~~omit~~

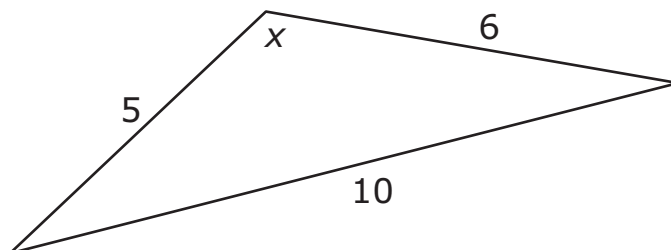
16 What is the end behavior of the function $f(x) = \frac{100}{1 + 5(0.75)^x}$?

- A $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = \infty$
- B $\lim_{x \rightarrow -\infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = 100$
- C $\lim_{x \rightarrow -\infty} f(x) = 1$ and $\lim_{x \rightarrow \infty} f(x) = \infty$
- D $\lim_{x \rightarrow -\infty} f(x) = 1$ and $\lim_{x \rightarrow \infty} f(x) = 100$

RELEASED



19 What is the **approximate** measure of angle x in the triangle below?



A 60.3°

B 80.4°

C 117.1°

D 130.5°

$$a^2 = b^2 + c^2 - 2(b)(c)\cos A$$

$$10^2 = 5^2 + 6^2 - 2(5)(6)\cos X$$

$$100 = 61 - 60\cos X \quad \rightarrow -0.65 = \cos X$$

$$39 = -60\cos X$$

$$\cos^{-1}(0.65) = X$$

$$130.54^\circ = X$$

20 The temperature, in degrees F, of the water in a large fish tank is modeled by the function $T(x) = \ln(1 + x) + 52.4$, where x is the number of pebbles in the tank.

Approximately how many pebbles are in the tank if the water is 58.3°F?

A 360

B 300

C 270

D 200

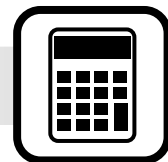
$$58.3 = \ln(1+x) + 52.4$$

$$5.9 = \ln(1+x)$$

$$e^{5.9} = 1+x$$

$$e^{5.9} - 1 = x$$

$$364.04 = x$$



21 A series is shown below.

$$1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots$$

$\overset{\times \frac{1}{5}}{\curvearrowright}$ $\overset{\times \frac{1}{5}}{\curvearrowright}$ $\overset{\times \frac{1}{5}}{\curvearrowright}$

Which statement is true about the sum of the series?

A The series converges to $\frac{7}{3}$.

$$S = \frac{a_1}{1-r} = \frac{1}{1-\frac{2}{5}}$$

B The series converges to $\frac{5}{2}$.

$$S = \frac{5}{3}$$

C The series converges to $\frac{5}{3}$.

~~D~~ The series diverges.

22 A circle is graphed using the parametric equations shown below.

$$x = 5\cos(t) + 3$$

$$y = 5\sin(t) - 1$$

Where is the center of the circle located?

A (-3, -1)

B (-3, 1)

C (3, -1)

D (3, 1)

~~omit~~

RELEASED