

Released Items

Student Name: \_\_\_\_\_

Fall 2015  
NC Final Exam  
**Advanced Functions and  
Modeling**



# Student Booklet



Public Schools of North Carolina  
State Board of Education  
Department of Public Instruction  
Raleigh, North Carolina 27699-6314



- 1 Suppose the function  $H(t) = 8.5\sin(0.017t - 1.35) + 12$  models the hours of sunlight for a town in Alaska, where  $t = 1$  is the first day of the year. Based on the function, what is the **approximate** range of daylight hours for the town?
- A 3.5 to 20.5
- B 4 to 20
- C 4.5 to 19.5
- D 5 to 19
- 2 The lifetime of a particular type of car tire is normally distributed. The mean lifetime is 50,000 miles, with a standard deviation of 5,000 miles. Of a random sample of 15,000 tires, how many of the tires are expected to last for between 45,000 and 55,000 miles?
- A 7,125
- B 10,200
- C 14,250
- D 14,850



- 3 The frequency table below shows the number of runners in specific age groups for a certain race.

Age Group	Number of Runners
0-10	
11-20	
21-30	
31-40	
41-50	
51-60	
61-70	
71-80	
81-90	

What is the shape of the distribution?

- A uniform
- B skewed right
- C skewed left
- D normal



- 4 A spinner labeled 1 to 9 gives each of the numbers 2, 5, 7, and 9 a  $\frac{3}{20}$  chance of being landed upon. The chance of landing on each of the other five numbers is equal. If the spinner is spun 1,000 times, which choice is the **most likely** outcome for the 1,000 spins?

A

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	110	112	111	111	109	112	112	111	112

B

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	82	148	78	80	149	79	151	81	152

C

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	120	122	100	103	108	126	113	104	104

D

<b>Number on Spinner</b>	1	2	3	4	5	6	7	8	9
<b>Number of Occurrences</b>	121	100	119	120	102	120	98	121	99



5 A group of 12 people need to form a line. The line will consist of exactly 9 of the people. Person X and Person Y have to be either third or fourth in line. How many different orders are possible?

A 79,833,600

B 1,209,600

C 604,800

D 362,880

6 The probability that it will rain on Saturday is  $\frac{2}{3}$ . The probability that the temperature on Saturday will reach  $100^{\circ}\text{F}$  is  $\frac{4}{9}$ . The probability that it will rain or reach  $100^{\circ}\text{F}$  on Saturday is  $\frac{4}{5}$ . What is the probability it will rain and reach  $100^{\circ}\text{F}$  on Saturday?

A  $\frac{14}{45}$

B  $\frac{16}{45}$

C  $\frac{24}{45}$

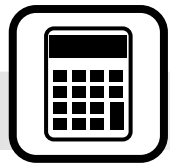
D  $\frac{26}{45}$



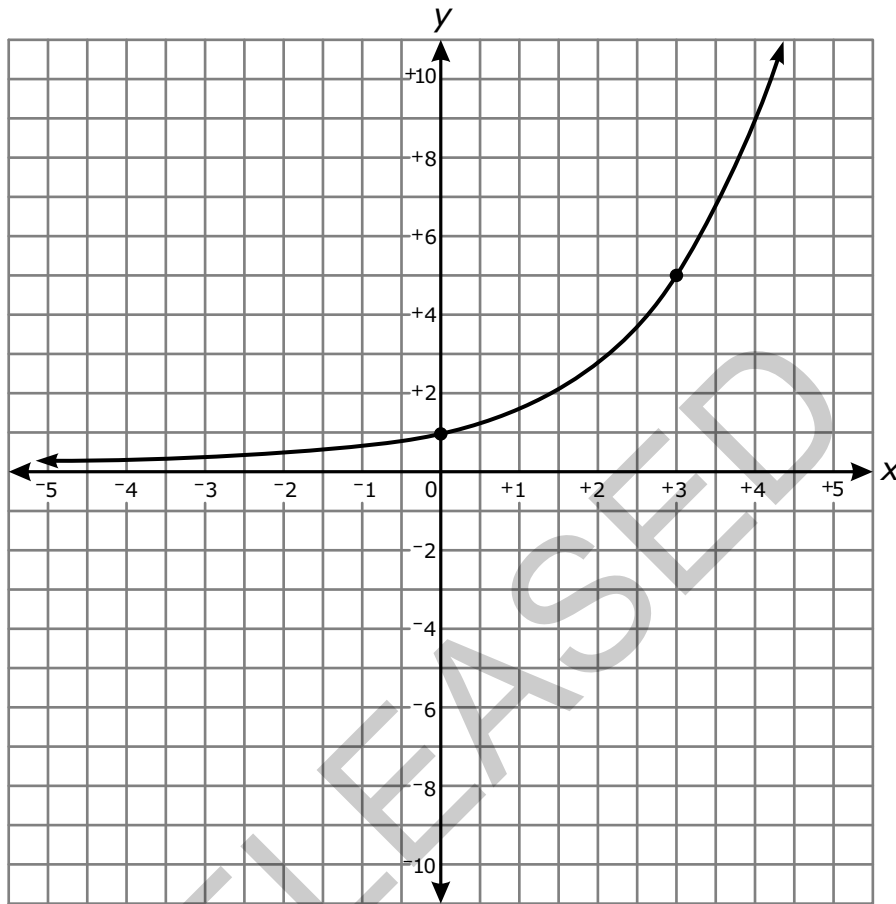
- 7 A manufacturing plant produces a special kind of lightbulb.
- Each lightbulb produced has a 0.040 probability of being defective.
  - Five lightbulbs are chosen at random from the production line.

To the nearest thousandth, what is the probability that exactly two of the five bulbs will be defective?

- A 0.014
- B 0.016
- C 0.018
- D 0.020
- 8 What is the meaning of the base of the function  $y = -\log(x)$ ?
- A As  $y$  decreases by 1,  $x$  increases by a factor of 10.
- B As  $y$  decreases by 1,  $x$  increases by 10.
- C As  $y$  increases by 1,  $x$  increases by a factor of 10.
- D As  $y$  increases by 1,  $x$  increases by 10.



9 The graph of  $y = a^x$  is shown below.



Which choice is closest to  $\log_a 3$ ?

- A 0.9
- B 2.1
- C 3.2
- D 4.8



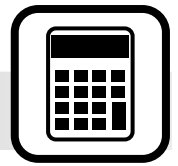
- 10 A piecewise function is shown below.

$$h(x) = \begin{cases} -2x^2 + 5x + 10 & \text{for } -4 \leq x < 3 \\ 2x + 3p & \text{for } 3 \leq x \leq 5 \end{cases}$$

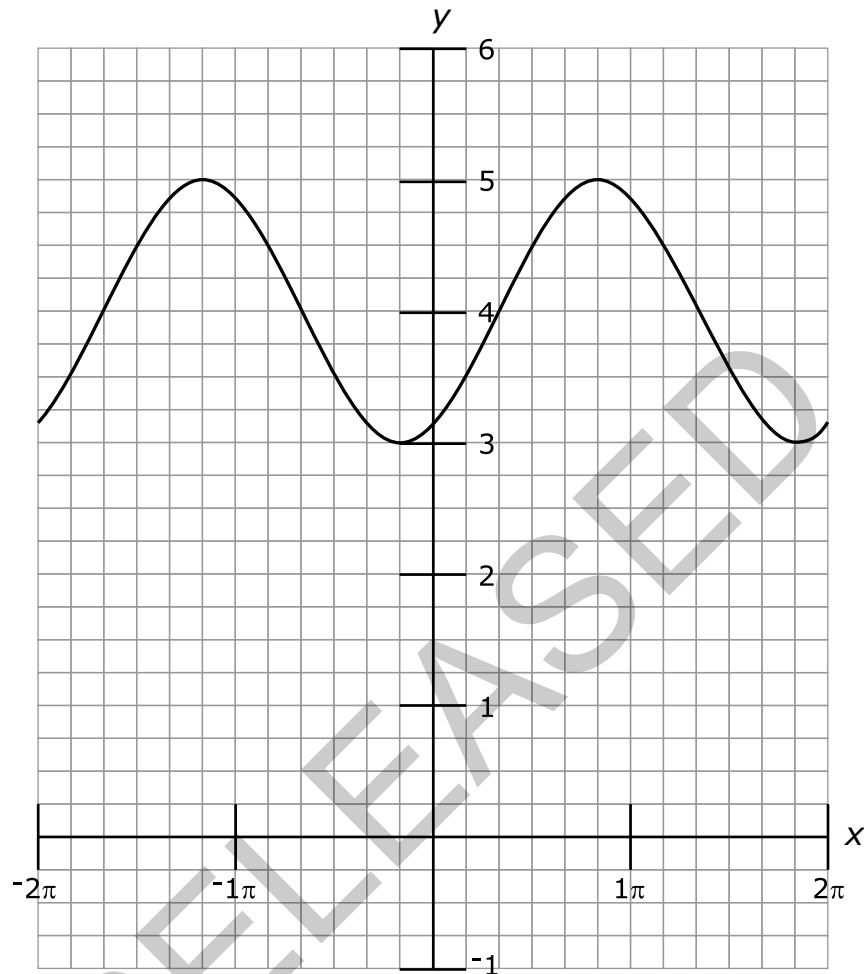
For what value of  $p$  will the function be continuous?

- A  $\frac{10}{3}$
- B  $\frac{1}{3}$
- C  $-\frac{25}{3}$
- D  $-\frac{34}{3}$
- 11 The equation  $y = 4.7x^{\frac{1}{6}}$  is graphed on the coordinate plane. How does increasing the denominator of the exponent transform the graph?
- A The transformed graph will approach a horizontal asymptote while the original graph will not.
- B The transformed graph will not approach a horizontal asymptote while the original graph will.
- C The transformed graph will go to  $\infty$  slower than the original graph as the value of  $x$  gets larger.
- D The transformed graph will go to  $\infty$  faster than the original graph as the value of  $x$  gets larger.





12 Which function correctly represents the graph below?



- A  $y = \sin\left(x - \frac{\pi}{3}\right) + 4$
- B  $y = \sin\left(x + \frac{\pi}{3}\right) + 4$
- C  $y = \cos\left(x - \frac{\pi}{3}\right) + 4$
- D  $y = \cos\left(x + \frac{\pi}{3}\right) + 4$



- 13 Which function has an amplitude that is twice the size and a period that is three times the size of the function  $y = 3 \cos\left(\frac{x}{4} - 1\right) + 2$ ?
- A  $y = 6 \sin\left(\frac{x}{12} - 3\right) + 1$
- B  $y = \frac{3}{2} \cos\left(\frac{3x}{4} + 1\right) - 3$
- C  $y = 6 \cos\left(\frac{3x}{4} - 1\right) + 3$
- D  $y = \frac{3}{2} \sin\left(\frac{x}{12} + 3\right) - 1$
- 14 A plane takes off and travels at an angle of  $40^\circ$  north of east at 110 mph for 2 hours. It then adjusts its path to head  $10^\circ$  west of north and travels in that direction for half an hour at a speed of 100 mph. **Approximately** how far away is the plane from its starting point?
- A 182 miles
- B 200 miles
- C 238 miles
- D 249 miles



- 15 Which statement is true about the fifth terms of the two sequences below?

$$a_n = 3n^2 - 6$$

$$b_n = 3(b_{n-1} - 6); b_1 = 10$$

- A The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 63.
  - B The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 63.
  - C The fifth term of the recursive sequence exceeds the fifth term of the explicit sequence by 21.
  - D The fifth term of the explicit sequence exceeds the fifth term of the recursive sequence by 21.
- 16 Which statement is true about the series shown below?

$$-4 + -2 + -1 + \frac{-1}{2} + \frac{-1}{4} + \dots$$

- A The series converges because  $|r| < 1$ .
- B The series diverges because  $|r| < 1$ .
- C The series converges because  $|r| > 1$ .
- D The series diverges because  $|r| > 1$ .



17 What is the explicit form of the equation  $a_n = a_{n-1} + 2(n - 1)$ ;  $a_1 = 1$ ?

A  $a_n = 2n - 1$

B  $a_n = n^2 - n + 1$

C  $a_n = n^2 - 2n + 2$

D  $a_n = 2n^2 - 2n - 1$

18 An investor bought 1,500 shares of a stock for \$6 a share. He estimates the probability that the stock will rise to a value of \$25 a share is 24%, and the probability it will fall to \$2 a share is 76%. What is the expected value of the investor's profit from buying the stock?

A \$13,560

B \$9,120

C \$6,720

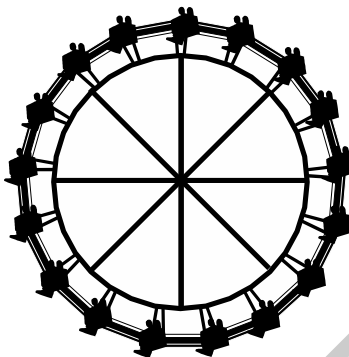
D \$2,280

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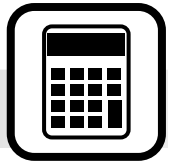
- 19 A Ferris wheel is designed in such a way that the height ( $h$ ), in feet, of the seat above the ground at any time,  $t$ , is modeled by the function

$$h(t) = 60 - 55 \sin\left(\frac{\pi}{10}t + \frac{\pi}{2}\right).$$



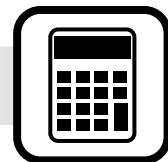
What is the **maximum** height a seat reaches?

- A 55 feet
  - B 60 feet
  - C 110 feet
  - D 115 feet
- 20 A teacher counts the final exam as 25% of each student's class grade. The remaining 75% is the mean of the student's test scores from the semester. Jaleesa's test scores for the semester are 86, 90, 92, and 80. What is the **minimum** score Jaleesa must get on the final exam to have a class grade of 85.0 or higher?
- A 77
  - B 79
  - C 81
  - D 83

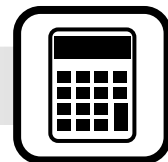


- 21 Two sides of a triangle measure 10 inches and 13 inches. The included angle between these sides is  $55^\circ$ . What is the **approximate** measure of the third side of the triangle?
- A 10.9 inches
  - B 11.2 inches
  - C 13.9 inches
  - D 16.2 inches
- 22 The third term of a geometric sequence is 96, and the fifth term is 1,536. What is the sum of the first ten terms of this sequence?
- A 4,092
  - B 1,572,864
  - C 2,097,150
  - D 33,554,400

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- 1 What transformations have occurred to create the function  $f(x) = 3x^3 - 4$  from the function  $g(x) = x^3$ ?
- A The graph of the function has been stretched horizontally and shifted up four units.
  - B The graph of the function has been stretched vertically and shifted up four units.
  - C The graph of the function has been stretched horizontally and shifted down four units.
  - D The graph of the function has been stretched vertically and shifted down four units.
- 2 An object is launched straight upward from ground level with an initial velocity of 50.0 feet per second. The height,  $h$  (in feet above ground level), of the object  $t$  seconds after the launch is given by the function  $h(t) = -16t^2 + 50t$ . At **approximately** what value of  $t$  will the object have a height of 28.0 feet and be traveling downward?
- A 2.39 seconds
  - B 1.84 seconds
  - C 1.56 seconds
  - D 0.73 seconds
- 3 What is the range of the function  $f(x) = -5 - 2(x + 3)^2$ ?
- A  $[-5, \infty)$
  - B  $(-\infty, 5]$
  - C  $(-\infty, -5]$
  - D  $(-\infty, \infty)$



~~omit~~ A wind that is blowing from the northwest toward the southeast can be represented by a vector. The vector has an eastward component and a southward component. If the eastward component has a magnitude of 5.00 miles per hour and the southward component has a magnitude of 15.00 miles per hour, in what direction is the wind blowing?

- A The wind is blowing in the direction  $71.6^\circ$  east of south.
- B The wind is blowing in the direction  $67.5^\circ$  east of south.
- C The wind is blowing in the direction  $22.5^\circ$  east of south.
- D The wind is blowing in the direction  $18.4^\circ$  east of south.

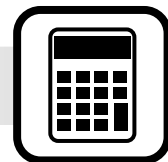
5 What value of  $x$  satisfies the equation  $\log_3(x - 4) = 2$ ?

- A 5
- B 10
- C 12
- D 13

6 A man is standing on level ground 50 feet away from the wall of a building. He looks up at a window on the building. The angle of elevation to the bottom of the window is  $28.5^\circ$ . He then looks up at the top of the building. The angle of elevation to the top of the building is  $35^\circ$ . What is the **approximate** distance between the bottom of the window and the top of the building?

- A 5.7 feet
- B 7.9 feet
- C 8.3 feet
- D 8.5 feet





~~X~~  
omit

Triangle  $WXY$  has the following properties:

- The angle at vertex  $W$  is  $14^\circ$ , and the angle at vertex  $X$  is obtuse.
- The side opposite vertex  $W$  has a length of 7.00 units.
- The side opposite vertex  $X$  has a length of 9.00 units.

What is the **approximate** length of the side opposite vertex  $Y$ ?

- A 1.73 units
- B 2.08 units
- C 3.26 units
- D 5.40 units

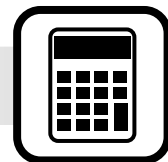
8 Consider these two trigonometric functions:

$$f(x) = 3 \sin(2x) + 4$$

$$g(x) = 3 \sin\left(2x - \frac{\pi}{2}\right) + 4$$

How should the graph of  $f$  be shifted to produce the graph of  $g$ ?

- A Shift the graph of  $f$  to the left  $\frac{\pi}{4}$  units to produce the graph of  $g$ .
- B Shift the graph of  $f$  to the right  $\frac{\pi}{4}$  units to produce the graph of  $g$ .
- C Shift the graph of  $f$  to the left  $\frac{\pi}{2}$  units to produce the graph of  $g$ .
- D Shift the graph of  $f$  to the right  $\frac{\pi}{2}$  units to produce the graph of  $g$ .



~~13~~  
omit

Two parametric equations are shown below, where  $t \geq 0$ .

$$x = \frac{1}{3}\sqrt{t} + 3$$

$$y = 4t^2 - 7$$

Which nonparametric equation can be used to graph the curve described by the parametric equations?

A  $y = \frac{4}{9}(x + 1) - 7$

B  $y = \frac{4}{3}(x + 3) - 7$

C  $y = 36(x - 1)^4 - 7$

D  $y = 324(x - 3)^4 - 7$

14 The formula for a sequence is shown below.

$$a_n = 2a_{n-1} + 3, a_1 = 3$$

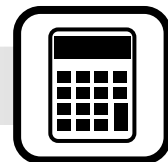
Which is another formula that represents the sequence?

A  $f(n) = 3(2^n - 1)$

B  $f(n) = 2n^3 - 3n^2 + 8n + 3$

C  $f(n) = 2(n^2 + 1)$

D  $f(n) = 3n^2 + 8n - 1$



15 When  $a_1 = 25,000$ , what is the sum of the infinite sequence defined by the equation  $a_{n+1} = 0.8a_n$ ?

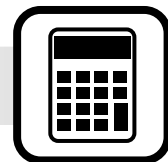
- A 125,000
- B 140,000
- C 160,000
- D 195,000

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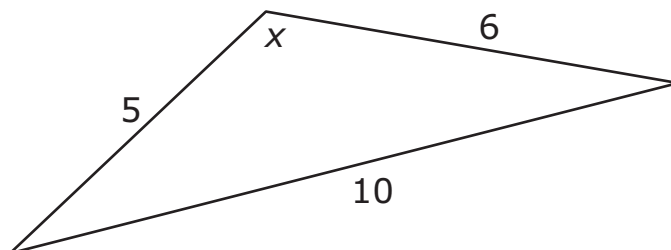
What is the end behavior of the function  $f(x) = \frac{100}{1 + 5(0.75)^x}$ ?

- A  $\lim_{x \rightarrow -\infty} f(x) = 0$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$
- B  $\lim_{x \rightarrow -\infty} f(x) = 0$  and  $\lim_{x \rightarrow \infty} f(x) = 100$
- C  $\lim_{x \rightarrow -\infty} f(x) = 1$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$
- D  $\lim_{x \rightarrow -\infty} f(x) = 1$  and  $\lim_{x \rightarrow \infty} f(x) = 100$

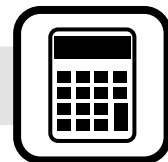
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- 19 What is the **approximate** measure of angle  $x$  in the triangle below?



- A 60.3°  
 B 80.4°  
 C 117.1°  
 D 130.5°
- 20 The temperature, in degrees F, of the water in a large fish tank is modeled by the function  $T(x) = \ln(1 + x) + 52.4$ , where  $x$  is the number of pebbles in the tank. **Approximately** how many pebbles are in the tank if the water is 58.3°F?
- A 360  
 B 300  
 C 270  
 D 200



21 A series is shown below.

$$1 + \frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \dots$$

Which statement is true about the sum of the series?

- A The series converges to  $\frac{7}{3}$ .
- B The series converges to  $\frac{5}{2}$ .
- C The series converges to  $\frac{5}{3}$ .
- D The series diverges.

~~22~~  
omit

A circle is graphed using the parametric equations shown below.

$$x = 5\cos(t) + 3$$

$$y = 5\sin(t) - 1$$

Where is the center of the circle located?

- A  $(-3, -1)$
- B  $(-3, 1)$
- C  $(3, -1)$
- D  $(3, 1)$