

Name: _____

Date: _____

Key

**EXPONENTIAL AND LOGARITHMIC REGRESSION
ALGEBRA 2 WITH TRIGONOMETRY**

Just as we fit data with a linear model in Unit #2, we can also fit data with exponential and logarithmic equations. Exponential regression is typically used on phenomena whose growth accelerates over time. Logarithmic regression is mostly used for phenomena that grow quickly at first and then slow down over time.

Exercise #1: The population of Jamestown has been recorded for selected years since 2000. The table below gives these populations.

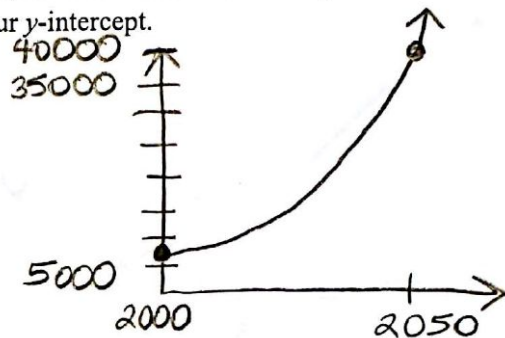
$(2000, 5158)$
 $(2050, 38751)$

Year	2002	2004	2005	2007	2009
Population	5564	6121	6300	6812	7422

- (a) Using your calculator, determine a best fit exponential equation, of the form $y = a \cdot b^x$, where x represents the number of years since 2000 and y represents the population. Round a to the nearest integer and b to the nearest thousandth.

$$y = 5158 \cdot (1.041)^x$$

- (b) Sketch a graph of the exponential function for the years 2000 to 2050. Label your window and your y-intercept.



- (c) By what percent does your exponential model predict the population is increasing per year? Explain.

4.1%

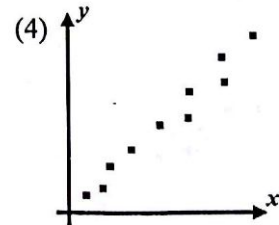
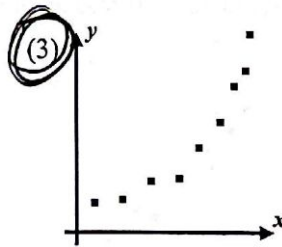
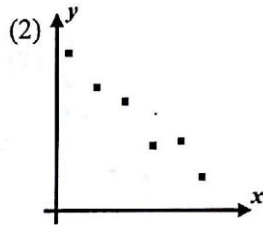
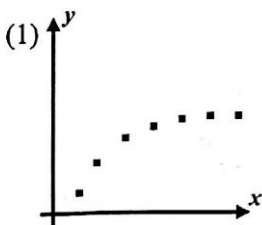
- (d) Algebraically determine the number of years, to the nearest year, for the population to reach 20 thousand.

$$20,000 = 5158(1.041)^x$$

$$3.8775 = (1.041)^x$$

$x \approx 33.7$ yrs

Exercise #2: Which of the following scatter plots would be best fit with an exponential equation?



Logarithmic regression models will be most applicable when a phenomenon grows very quickly at first, but then its growth rate slows down over time. Depending on your graphing calculator, the regression might occur with different bases of logarithms. We will assume a **natural logarithm** model.

Exercise #3: A corn plant will grow rapidly after it first emerges from the soil and then eventually slows its growth rate. Agronomists record the average height for a particular varietal of corn as a function of the number of days since it was planted. The data is given in the table below.

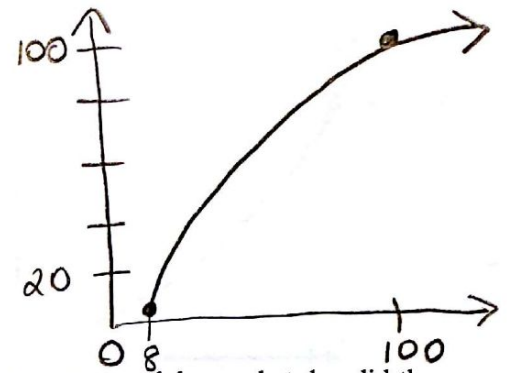
Days	8	14	22	40	48	54
Height (in)	4	18	48	60	71	73

(100, 97)
(8, 4)

- (a) Use your calculator to determine the best fit logarithmic equation for this data set. Express your answer in the form $y = a + b \ln x$, where x represents the days since planting and y represents the average height. Round both a and b to the nearest integer.

$$y = -74 + 37 \ln x$$

- (b) Sketch a graph of your equation below for $0 \leq x \leq 100$. Label your window.



- (c) The corn will develop tassels when it reaches a height of four feet. To the nearest day, use your model to predict when the corn will tassel. 48 in

$$48 = -74 + 37 \ln x$$

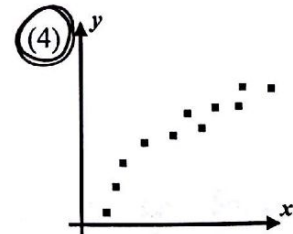
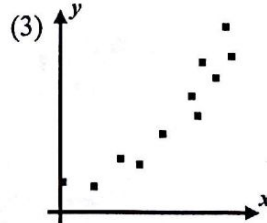
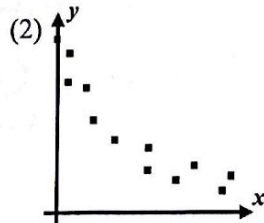
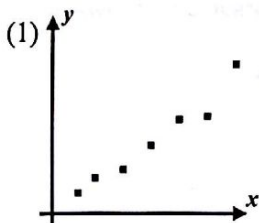
27 days
or 22

- (d) According to your model, on what day did the corn germinate (emerge from the ground)? Round to the nearest day.

$$0 = -74 + 37 \ln x$$

Day 7

Exercise #4: Which of the following scatter plots would be best fit with a logarithmic equation?



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**EXPONENTIAL AND LOGARITHMIC REGRESSION
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK**

APPLICATIONS

1. Rabbits were accidentally introduced to an island where their population is growing rapidly. Biologists studying the rabbits have periodically recorded their population since they were introduced to the island. The data they took is shown below.

Years Since Introduction, x	2	5	7	11	15
Population of Rabbits, y	75	100	112	205	290

- (a) Determine an exponential regression equation, in the form $y = a \cdot b^x$, that models this data. Round a to the *tenth* and b to the *hundredth*.

$$y = 58.19 (1.11)^x$$

- (c) Based on your model in part (a), by what percent is the rabbit population growing each year?

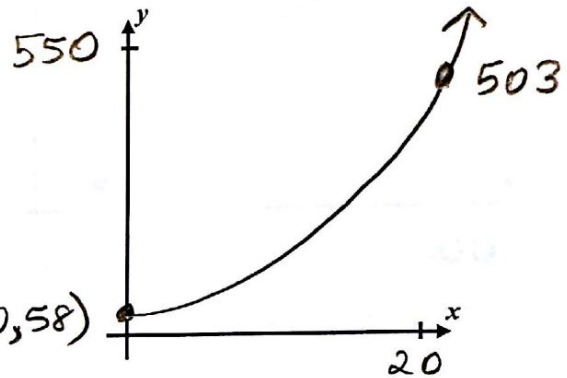
$$11\%$$

- (d) Graphically determine, to the nearest *tenth* of a year, when the rabbit population will reach 350.

$$350 = 58.19 (1.11)^x$$

$$x = 17.2 \text{ years}$$

- (b) Sketch a graph of the rabbit population below on the axes provided for $0 \leq x \leq 20$. Label your graphing window and your y -intercept.



2. The infiltration rate of a soil is the number of inches of water per hour it can absorb. Hydrologists studied one particular soil and found its infiltration rate decreases exponentially as a rainfall continues.

Time, t (hours)	0	1.5	3.0	4.5	6.0
Infiltration Rate, I (inches per hour)	5.3	3.1	2.4	1.6	0.7

Create an exponential model that best fits this data set. Round coefficients to the nearest *hundredth*. Use your model to algebraically determine the time until the rate reaches 0.25 inches per hour. Round your answer to the nearest *tenth* of an hour.

$$y = 5.47 (.73)^x$$

$$0.25 = 5.47 (.73)^x$$

$$x = 9.8 \text{ hrs}$$



During prolonged cold in northern latitudes, thick ice will grow on lakes. A particular lake in Ladysmith, Wisconsin, has its ice thickness measured every day after the temperature fell below zero. The data is shown in the table below.

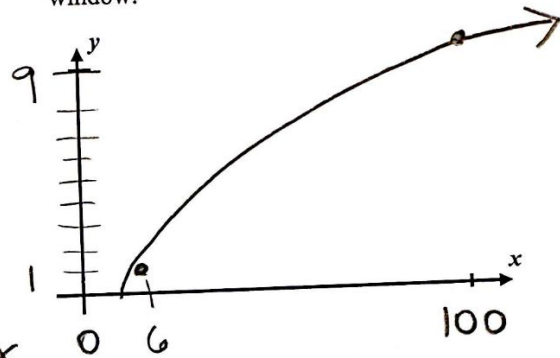
Days Below Freezing, x	6	9	14	20	27	32	55
Ice Thickness, y (inches)	0.8	1.7	3.5	4.0	5.2	5.5	7.2

$(6, 0.8)$
 $(100, 8.9)$

- (a) Find a logarithmic equation, of the form $y = a + b \ln x$, that best fits this data set. Round both coefficients to the nearest tenth.

$$y = -4.5 + 2.9 \ln x$$

- (b) Create a sketch of this function over the interval for the first hundred days the temperature is below freezing. Label your window.



- (c) Use a table on your calculator to determine, to the nearest day, the number of days below freezing necessary for the ice to reach a thickness of one foot. Provide numerical evidence to the nearest tenth of a day.

12 in

$$12 = -4.5 + 2.9 \ln x$$

$$x \approx 295.8 \text{ days}$$

4. In the table below, the year, y , in which a certain population, x , was reached by Charleston, Illinois is given.

Population, x	5,000	6,000	7,000	8,000	9,000
Year, y	1972	1980	1984	1987	1989

Find a logarithmic model, of the form $y = a + b \ln x$, that best fits this data. Round your coefficients to the nearest integer. Using your model, algebraically determine the population, to the nearest whole number, of Charleston in the year 2010.

$$y = 1731 + 28 \ln x$$

$$2010 = 1731 + 28 \ln x$$

$$21,254$$

