Key

Exponential Functions

$y = ab^x$

Exponential functions are when a variable is in the spot of the exponent.

Example: $y = 2(3)^x$

Example: I invested $1,000 at an interest rate of 5% per year compounded annually.

At the end of year:	Computation:	Amount I have:
1	1,000(1.05)	$1,050
2	1,050(1.05) = 1,000(1.05)(1.05)	$1,102.50
3	1,102.5(1.05) = 1,000(1.05)(1.05)(1.05)	$1,157.625

Equation:

In the equation $y = ab^x$, a is the initial amount (in our case $1,000) and b is the growth or decay factor (in our case it is growth - a growth factor of 1.05). If the b is larger than 1 then you have a growth function. If the b is between 0 and 1 then you have a decay function.

To find **growth factor** take 1 and add to your percent growth (1 + .05 = 1.05)
To find the **decay factor** take 1 and subtract your percent decay (1 - .25 = .75)

Key words in exponentials - if something increases or decreases by a percent, doubling (increases by 100%), halving (decrease by 50%) tripling, etc...

1) The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%. (a) Find the growth factor for the world's population. (b) Suppose the rate of increase continues to be 1.26%. Write a function to model world population growth.

a) .0126 + 1 = 1.0126

b) $y = 6.08(1.0126)^x$

2) A computer valued at $6500 depreciates at the rate of 14.3% per year. (a)

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Write a function that models the value of the computer. (b) Find the value of the computer after three years.

a) $y = 6500(1 - .143)^x$

b) $y = 6500(1 - .143)^3 = \$4091$

3) The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying. (a) Write a function that models the change in the animal population. (b) Estimate the number of years until the population first drops below 15 animals.

a) $y = 80(1 - .035)^x$

b) $15 = 80(1 - .035)^x$

$.1875 = .965^x$

about 47 years

4) The value of an industrial machine decreases 25% per year. After six years, the machine is worth \$7500. What was the original value of the machine?

$7500 = a(1 - .25)^6$

$a = \$42{,}140$

5) The function $y = 20(0.975)^x$ models the intensity of sunlight beneath the surface of the ocean. The output y represents the percent of surface sunlight intensity that reaches a depth of x feet. The model is accurate from about 20 feet to about 600 feet beneath the surface. (a) Find the percent of sunlight 50 feet beneath the surface of the ocean. (b) Find the percent of sunlight at a depth of 370 ft.

a) $y = 20(0.975)^{50} = 5.64\%$

b) $y = 20(0.975)^{370} = .0017\%$

expreg

Write an exponential function $y = ab^x$ for a graph that includes the given points.

6) (4, 8), (6, 32)

$y = .5(2)^x$

7) (-3, 24), (-2, 12)

$y = 3(.5)^x$

Half Life

$y = a(.5)^{(\text{time}/\text{half-life})}$

The half life of a substance is the time it takes for half of the material to decay

8) A 3000-mg sample of a certain radioactive element has a half life of 3 seconds. How much of the sample remains after 1 minute?

$y = 3000(.5)^{60/3}$

$.0029$ mg

9) Arsenic-74 is used to locate brain tumors. It has a half life of 17.5 days. Write the exponential decay function of a 90-mg sample. Use the function to find the amount remaining after 6 days.

$y = 90(.5)^{6/17.5}$

$y = 70.96$ mg

10) Phosphorus-32 is used to study a plant's use of fertilizer. It has a half life of 14.3 days. Write the exponential decay function for a 50-mg sample. Find the amount of phosporus-32 remaining after 84 days.

$y = 50(.5)^{84/14.3}$

$y = .85$ mg

11) Iodine-131 is used to find leaks in water pipes. It has a half-life of 8.14 days. Write the exponential decay function for a 200-mg sample. Find the amount of iodine-131 remaining after 72 days.

$y = 200(.5)^{72/8.14}$

$y = .43$ mg

$3 \cdot \log_7 7$

$3 \cdot 1$

Compound Interest

The compound interest formula for the amount A in an account is $A = P\left(1 + \frac{r}{n}\right)^{nt}$

P = principle

r = rate (decimal)

n = # times compounded

t = time

12) Jodie's parents started a savings account for her when she was born. They invested

$500 in an account that pays 6% interest compounded annually. Find the balance of the account after each of the first three years.

$$A = 500\left(1 + \frac{.06}{1}\right)^{(1 \cdot 3)}$$

$$= \$ 596$$

1 yr = 530
2 yrs = 562
3 yrs = 596

13) Graham's grandparents started a savings account for him when he was born. They invested $100 in an account with 8% annual interest compounded quarterly. How much is in his account on his 16th birthday?

$$= 100\left(1 + \frac{.08}{4}\right)^{(4 \cdot 16)}$$

$$= \$355$$

Interest Compounded Continuously

The formula for continuously compounded interest for the amount A in and account is

$A = Pe^{rt}$

14) Suppose your ancestor deposited $5 in an account with an annual interest rate of 3.5% compounded continuously. If the money was first deposited 200 years ago, what is the value of the account today?

$$A = 5e^{(.035)(200)}$$

$ 5483

15) Suppose you invest $1050 at an annual interest rate of 5.5% compounded continuously. Find the amount in the account after 5 years. How long will we need to leave the money in the account in order to double our investment?

$$A = 1050(e)^{(.055)(5)}$$

$$A = \$1382$$

$$2100 = 1050 \cdot e^{.055t}$$

$$2 = e^{.055t}$$

y = intersect

about 12.6 yrs

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AFM
Word Problems

Name Key

1. Mr. and Mrs. Sauls are planning to take a cruise for their 25th wedding anniversary. They have six years to save \$3500 for the cruise. If the six-year certificate of deposit they buy now pays 6% compounded continuously, how much should they invest now in order to have \$3500 for the cruise? $3500 = Pe^{(.06)(6)}$

$P = \$2442$

2. An initial investment of \$100 is now \$149.18. The interest rate is 8% compounded continuously. How long has the money been invested?

$149.18 = 100e^{(.08)(t)}$ $t = 5$ yrs

$1.4918 = e^{.08t}$

$.1737 = .08t \cdot \log e$

3. Suppose \$250 is deposited in a savings account. The interest rate is 10% compounded continuously. When will the original deposit be doubled?

$500 = 250e^{(.1)(t)}$ $\log 2 = .1t\log e$

$2 = e^{.1t}$ $t = 6.9$ yrs

4. A piece of machinery valued at \$250,000 depreciates at 12% per year by the fixed rate method. After how many years will the value have depreciated to \$100,000? $100{,}000 = 250{,}000(1-.12)^t$

$.4 = .88^t$ $t = 7.17$ yrs

5. The Smith family bought a house for \$80,000. The house is now worth \$140,000. Assuming a steady rate of growth of 5.75%, about how long ago did the family buy the house? $140{,}000 = 80{,}000(1+.0575)^t$

$1.75 = 1.0575^t$ $t = 10$ yrs ago

6. The function $f(x) = 15{,}557 + 5259\ln x$ models the average cost of a new car, $f(x)$, in dollars, x years after 1989. When was the average cost of a new car \$25,000? $25{,}000 = 15{,}557 + 5259\ln x$

$1.7956 = \ln x$ $9443 = 5259\ln x$

$\boxed{1995}$ $x = 6$ yrs

7. The formula $A = 15.9e^{0.0235t}$ models the population of Florida, A, in millions, t years after 2000. When will the population of Florida reach 17.5 million? $17.5 = 15.9e^{.0235t}$

$1.1006 = e^{.0235t}$ $t = 4$ yrs $\boxed{2004}$

8. The formula $A = P\left(1+\frac{r}{n}\right)^{nt}$ describes the accumulated value, A, of a sum of money, P, the principal, after t years at an annual percentage rate r compounded n times a year. How long will it take \$25,000 to grow to \$500,000 at 9% annual interest rate compounded monthly?

$500{,}000 = 25{,}000\left(1+\frac{.09}{12}\right)^{12 \cdot t}$

$20 = (1.0075)^{12t}$ 33 yrs

9. How long, to the nearest tenth of a year, will it take \$1000 to grow to \$3600 at 8% annual interest compounded quarterly?

$3600 = 1000\left(1+\frac{.08}{4}\right)^{4 \cdot t}$ 16.2 yrs

$3.6 = 1.02^{4t}$

10. The formula $P = 50e^{\frac{-t}{25}}$ gives the power output P, in watts, available to run a certain satellite for t days. Find how long a satellite with the given power output of 10 W will operate. $10 = 50e^{-t/25}$

$.2 = e^{-t/25}$ 40 days

Day 8

Key

Applications of Logarithms

1. Find the time period required for $7000 invested at 10% compounded semi-annually to grow to 10,000.

$$10{,}000 = 7{,}000\left(1+\frac{.1}{2}\right)^{2\cdot t}$$

$$1.428 = (1.05)^{2t}$$

$$\log 1.428 = 2t \cdot \log 1.05$$

$t \approx 3.65$ yrs

2. The value of an investment is given by $f(x) = 237.50(1.052)^x$, where x is the number of 6-month periods. Find the number of complete periods until the investment is worth at least $600.

$$600 = 237.50(1.052)^x$$

$$2.526 = 1.052^x$$

$$\log 2.526 = x\log 1.052$$

18.28 or 19 periods

omit

3. Cell population doubles every 3 h. How long would it take 4 cells to reach a count of 16,384?

grows 100% $\quad 1+1=2$ $\quad b=$

$16384 = 4\cdot 2^x$

$x = 12$ cycles

$\times 3$ hrs

	hrs
4	3 → 8
8	6 → 16
16	9 → 32
32	12 → 64
64	15 → 128
128	18 → 256

$$16384 = 4\cdot(2)^{x/3}$$

$$4096 = 2^{x/3}$$

$$\log 4096 = \frac{x}{3}\log 2$$

36 hrs

4. For every meter below the water surface, light intensity is reduced by 5%. At what depth is light intensity 40% of that at the surface?

$1 - .05 = .95$

omit

$$40 = 100(1-.05)^x$$

$$.4 = .95^x$$

$x \approx 17.86$ m