

Key

Exponential Functions

Exponential Model: $y = ab^x$ $a = \frac{\text{initial amount}}{\text{amount}}$ $b = \frac{\text{growth/decay factor}}{\text{factor}}$
 Exponential Growth: $b = 1+r$ Exponential Decay: $b = 1-r$
 $r = \text{rate (decimal)}$

1) The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%. (a) Find the growth factor for the world's population. (b) Suppose the rate of increase continues to be 1.26%. Write a function to model world population growth.

a) $0.0126 + 1 = 1.0126$
 b) $y = 6.08(1.0126)^x$

2) A computer valued at \$6500 depreciates at the rate of 14.3% per year. (a) Write a function that models the value of the computer. (b) Find the value of the computer after three years.

a) $y = 6500(1 - 0.143)^x$
 b) $y = 6500(1 - 0.143)^3 = \4091

3) The value of an industrial machine decreases 25% per year. After six years, the machine is worth \$7500. What was the original value of the machine?

$7500 = a(1 - 0.25)^6$
 $a = \$42,140$

Half Life: $y = a \cdot \left(\frac{1}{2}\right)^{\text{time}/HL}$

The half life of a substance is the time it takes for half of the material to decay

4) A 3000-mg sample of a certain radioactive element has a half life of 3 seconds. How much of the sample remains after 1 minute?

$y = 3000(.5)^{60/3}$
 $.0029 \text{ mg}$

5) Arsenic-74 is used to locate brain tumors. It has a half life of 17.5 days. Write the exponential decay function of a 90-mg sample. Use the function to find the amount remaining after 6 days.

$y = 90(.5)^{6/17.5}$
 70.96 mg

6) Phosphorus-32 is used to study a plant's use of fertilizer. It has a half life of 14.3 days. Write the exponential decay function for a 50-mg sample. Find the amount of phosphorus-32 remaining after 84 days.

$y = 50(.5)^{84/14.3}$
 $.85 \text{ mg}$

Compound Interest:

The compound interest formula for the amount A in an account is $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = \text{principle}$ $r = \text{rate (decimal)}$

$n = \frac{\# \text{ times}}{\text{compounded}}$ $t = \text{time}$

7) Jodie's parents started a savings account for her when she was born. They invested \$500 in an account that pays 6% interest compounded annually. Find the balance of the account after three years.

$A = 500\left(1 + \frac{0.06}{1}\right)^{1 \cdot 3}$
 $A = \$596$

8) Graham's grandparents started a savings account for him when he was born. They invested \$100 in an account with 8% annual interest compounded quarterly. How much is in his account on his 16th birthday?

$A = 100\left(1 + \frac{0.08}{4}\right)^{4 \cdot 16}$
 $A = \$355$

Interest Compounded Continuously: $A = Pe^{rt}$

9) Suppose your ancestor deposited \$5 in an account with an annual interest rate of 3.5% compounded continuously. If the money was first deposited 200 years ago, what is the value of the account today?

$A = 5e^{(.035)(200)} = \$5483$

10) Suppose you invest \$1050 at an annual interest rate of 5.5% compounded continuously. Find the amount in the account after 5 years.

$A = 1050e^{(.055)(5)}$
 $= \$1382$